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PIM ADANG

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door

Petrus Johannes Maria Adang

geboren te Waalwijk

Promotor: Prof. Dr. Ir. A. Kapteyn

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Pim Adang
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CHAPTER 1

INTRODUCTION

1. Prelude.

Of the many activities of mankind, consuming is probably the aspect which has been most extensively investigated by economists and econometricians: The consumption in a national economy has been studied, as well as the consumption of individual consumers. The total consumption of a consumer has been considered, as well as the consumption of particular commodities. Short term movements have received attention, as well as long term developments.

Given these many different aspects, it is clear that any study dealing with consumption can only cover a small number of the topics which are worth to be investigated. The study at hand concentrates on the behaviour of individual consumers. Stated more precisely, the study focuses attention on a framework which is frequently used to model consumer behaviour, the so-called life cycle model under uncertainty.

The aim of this introduction is twofold. In the first part, a brief overview of the most important developments of the life cycle theory is given. For a more extensive review the reader is referred to the surveys of, for instance, Blundell (1987, 1988). In the second part, those aspects of the life cycle model which will be considered in this study, are introduced.

2. A brief history of life cycle theory.

Starting-point for many overviews of this branch of economic theory is the seminal paper by Modigliani and Brumberg (1954), although elements of this theory can already be found in work by, amongst others, Ramsey (1928). The central tenet of the life cycle theory is that individuals, when deciding on their consumption for a particular period, take both the desired future consumption levels, as well as the future development of variables influencing the consumption decisions, like income and prices, into account. In their study, Modigliani and Brumberg formulated a model based on this premise, and confronted the implications which could be derived from this model with empirical findings. By demonstrating that it was possible to use a life cycle model in this way, and because of the potential usefulness of this model in many research areas, Modigliani and Brumberg's study motivated many researchers to redirect attention from static and short-term models, to this long-term and dynamic framework.

Apart from the life cycle hypothesis stated above, Modigliani and Brumberg introduced some additional assumptions in order to arrive at the model from which they derived their conclusions. In later years, many of these additional assumptions came under close scrutiny. Especially the assumption that consumers are exactly informed about all future events which can possibly influence their consumption, the so-called perfect foresight assumption, was considered by many to be untenable.

Therefore, subsequent studies tried to relax this assumption by allowing variables pertaining to future periods to be uncertain. Uncertain variables are, in line with the approach usually employed in life cycle studies, defined as variables whose actual values are not known by the consumer at the moment he or she chooses the consumption path for the (remaining) lifetime, but the probability distribution of these variables is known. The most popular way of incorporating this uncertainty in the life cycle model was by introducing the well-known expected utility hypothesis (see, for example, Sinn (1983), chapter 2.c).

This, despite the objections which were raised against this hypothesis. Machina (1987), for instance, reports a number of experiments contradicting the expected utility hypothesis. However, these experiments often are somewhat artificial in the sense that they are usually concerned with choices individuals make when offered cleverly constructed bets, but not with considerations individuals make when they have to decide on 'real world' problems. Hence, the outcome of these experiments does not automatically imply that the expected utility hypothesis must be rejected as a suitable framework for modelling the actual behaviour of consumers.

A possible explanation for the popularity of this hypothesis could lie in the ease with which models based on the expected utility hypothesis can be applied in empirical work. This easy applicability is mainly due to Hall, who in his 1978 paper derived necessary conditions for a consumption plan to be optimal, in the sense that it results in the maximum expected lifetime utility. Until Hall's contribution, researchers were forced to make ad hoc assumptions in order to deal with the fact that many variables influencing the consumption plan, like future incomes and prices, are uncertain. By taking this uncertainty, which is inherent in models incorporating future events and choices, as his starting-point, and by applying a calculus of variations argument, he arrived at remarkably simple necessary conditions, which could easily be checked in empirical applications.[1]

Later on, alternative techniques were used for solving the life cycle model under uncertainty. For example, MaCurdy (1983) used a dynamic programming formulation of the model in order to derive conditions which must hold at the optimum, whereas Melenberg and Alessie (1989) applied a generalized Lagrange Multiplier rule to derive such conditions. This latter approach especially seems promising, as it enables the estimation of quite general life cycle models in a rather straightforward way.

3. Issues to be addressed.

The present study follows Modigliani and Brumberg (1954), and considers the life cycle model at the micro level. That is, the model is used in order to offer a description and an explanation of the behaviour of individual consumers. The relevance of the model at a more aggregated level is not considered. This despite the fact that many studies either are concerned with the question whether the life cycle hypothesis holds at the macro level, or use the hypothesis as a building block of a macro model. However, the appropriateness of this approach is called into question by some researchers. For example, Blundell (1988), MaCurdy (1987) and Deaton (1987) all stress the severity of restrictions, like the representative consumer assumption, which must be imposed to justify the use of the life cycle hypothesis at the macro level. Moreover, the latter two authors show that under quite plausible circumstances, for instance, the presence of corner solutions, the evolution of macro quantities does not have to correspond with predictions based on the representative consumer model. Because of these drawbacks, attention in this study will be restricted to the life cycle model at the micro level.

In particular, consumption disaggregated in a number of commodity categories will be considered. Thus, this study follows many other contributions, in assuming that not only the consumer's decisions regarding total consumption can be adequately modelled by the life cycle model, but also the decisions on the composition of the consumption. In the empirical illustrations presented in this study, only the two-category case is considered, but the generalization to more categories is straightforward.

The first aspect of this multi-good life cycle model under uncertainty which will be considered, is its stochastic specification. Although it is used in many studies, a straightforward multi-good version of Hall's (1978) model has its difficulties. In particular, the first order conditions which must hold at the optimum do not only result in the well-known intertemporal Euler equations, but also in intratemporal relations between the marginal utilities of the different goods. These intratemporal relations are deterministic, implying that they should hold exactly for each observation in the data set used for a particular empirical application.

However, it is very unlikely that one can develop a model which meets this requirement, and which can actually be estimated. As this indicates that straightforward multi-good versions of Hall's (1978) model are likely to be misspecified, it needs to be modified. Put more precisely, the model must be changed in such a way that the intratemporal relations become non-deterministic.

Several ways of achieving this aim are discussed in chapter 2. Apart form an ad hoc approach, simply adding error terms to the intratemporal equations without indicating how they might fit in the model, two other approaches are suggested in the literature: including either measurement errors or random preferences in the life cycle model. A main disadvantage of these two approaches is that they usually require additional (often restrictive) assumptions to enable estimation. Especially since one of the attractive features of Hall's (1978) approach is that one can estimate the model on the basis of relatively few assumptions, one would rather do without these additional ones.

Therefore, an alternative approach, suggested by Melenberg and Alessie (1989), is considered. They tackle the problem of deterministic intratemporal equations by introducing intratemporal uncertainty, which can be regarded as the within period analogue of the familiar intertemporal uncertainty. That is, because it is assumed by Melenberg and Alessie (1989) that a consumer does not only plan the consumption across periods, but also the consumption in each period, the consumer is confronted with uncertainty within each period. As a result, the intratemporal relations become non-deterministic. In chapter 2, the presence of this intratemporal uncertainty is motivated, and some life cycle models incorporating this form of uncertainty are estimated and tested.

Chapter 3 is concerned with another phenomenon with which one can be confronted when studying disaggregated consumption, namely that the consumption of some goods display a strongly fluctuating pattern. More in particular, some goods are either not consumed at all, or in relatively large quantities only. An example of such a good is the vacation of households.

It is argued in chapter 3 that one cannot fully explain such a consumption pattern within the life cycle models which are usually considered (i.e., the type of models studied in chapter 2). Therefore, a modification of the multi-good version of Hall's (1978) model is put forward. The objective of this modification is to change the model in such a way, that choosing to consume a (relatively) small positive quantity of a certain good can never be optimal. This is achieved by making either the preference ordering, or the budget constraint non-convex for these small values, by introducing a proper transformation in the life cycle model.

In chapter 3, such a transformation is implemented in the model considered in chapter 2, and the consequences of the modification are studied. Subsequently, this modified model is estimated in order to assess the relevance of the transformation for a particular case: the consumption of vacation.

In chapter 4, a problem which one is likely to encounter if one tries to employ a life cycle model in empirical work is taken up. This problem stems from the fact that such models are concerned with consumption, whereas data sets usually only contain information on expenditures. The difference between these two quantities can be considerable, especially if the data are collected during a (relatively) short reporting period, and refer to a number of commodity categories.

In the literature, several ways of dealing with this difference are suggested. A well-known approach is put forward in the so-called 'infrequency of purchase' literature, which focuses on the fact that (unobserved) consuming and (observed) purchasing occur at different frequencies. This implies that, in order to be able to use the expenditure data for the analysis of consumer behaviour, a link between the two quantities must be established. This is usually done by defining the consumption variable as the quotient of the expenditures over the so-called purchase probability. This probability is defined as the chance that a purchase is observed during the reporting period. A disadvantage of this link is that it is static, and not easily made dynamic.

Therefore a second approach to be found in the literature, which was developed with the explicit aim to establish such a dynamic link in order to deal with the durable nature of many goods, is considered. However, the links introduced in this kind of study are usually not very flexible. Therefore, a more general framework, which nests this second type of models, is introduced in chapter 4.

It is shown that in order to estimate this model, both consumption and expenditure data are required. One can, under certain assumptions, express the model in expenditure terms only, which would imply that estimation of the model on the basis of expenditure data is posssible. However, the model one ends up with requires, in general, so much information on the expenditure plans of consumers (covering the complete lifetime) to enable estimation, that this offers no real alternative.

Finally, chapter 5 reiterates the main findings of this study.

Note to chapter 1.

1 Empirical applications, of course, require further assumptions regarding the specification of, for instance, the utility function, which are of an ad hoc nature.

CHAPTER 2

INTRATEMPORAL UNCERTAINTY IN THE MULTI-GOOD LIFE CYCLE CONSUMPTION MODEL: MOTIVATION AND APPLICATION

1. Introduction.

Since Hall (1978) many economists have studied consumer behaviour under uncertainty within the context of the life cycle hypothesis by means of Euler equations. The standard life cycle hypothesis states that a consumer decides in each period on (total) consumption by maximizing an intertemporally additive (von Neumann-Morgenstern) expected utility function subject to a lifetime wealth budget constraint. From the first order conditions of this optimization problem one can derive Euler equations, which have an attractively simple form: the marginal utility of consumption evolves according to a random walk with trend. By using the Euler equations, the model can be estimated by the Generalized Method of Moments, as proposed by Hansen and Singleton (1982).

If Hall's (1978) life cycle model is extended to deal with more than one good per period, the first order conditions that should hold at the optimum not only result in intertemporal Euler equations, but also imply deterministic intratemporal relations between the marginal utilities of the different goods. The deterministic nature of these intratemporal relations has serious consequences for empirical applications of this model: the intratemporal relations must hold *exactly* for each observation in the data set used for the particular application. As it is very unlikely, or even impossible, that this requirement will be met, the presence of such deterministic relations indicates some form of misspecification.

In order to overcome this misspecification, the multi-good version of Hall's (1978) model needs to be modified. Several modifications suggested in the literature are discussed in section 2. Since the best-known solutions, like incorporating random preferences or measurement errors, have serious disadvantages, an alternative approach recently suggested by Melenberg and Alessie (1989) is used. In order to make the intratemporal equations non-deterministic, they introduce additional uncertainty into the life cycle model, which can be interpreted as intratemporal uncertainty, as opposed to the already existing intertemporal uncertainty. Since Melenberg and Alessie (1989) concentrate on the technical aspects and hardly motivate the presence of additional intratemporal uncertainty in the multi-good life cycle consumption model, such a motivation is given in section 2.

The consequences of incorporating intratemporal uncertainty in the multi-good life cycle consumption model are studied in section 3. In particular, attention is paid to the way in which the first order conditions characterizing the optimal consumption path can be combined into a system of moment restrictions, which can be used for testing and estimation.

In section 4, the estimation and testing results of some (relatively simple) two good versions of the life cycle consumption model with intratemporal uncertainty are presented. For this, a Dutch panel containing information on the monthly expenditures on several commodity categories is used. Since this panel contains many households per period and also has a large time dimension, whereas at the same time there are not so many variables that vary over households, it will be assumed that averaging over both households and time is allowed in applying the Generalized Methods of Moments. Notice, however, that averaging over time includes as an assumption that the underlying population is stationary over time (in some sense), cf. Hansen (1982). The estimates are, generally speaking, in accordance with consumer theory. The test results imply that all but one of the versions incorporating intratemporal uncertainty are not rejected.

2. Intratemporal uncertainty.

A commonly used approach to the estimation of a multi-good life cycle consumption model is to apply a two step procedure, corresponding to two stage budgeting. The first step consists of estimating an intratemporal demand system which can be obtained by maximizing the intratemporal utility function subject to the intratemporal budget constraint (i.e. the second stage of the consumer's optimization problem). The second step uses the results of the first step for the estimation of the equation which sets the expected marginal utilities of money in different periods equal to one another. This so-called Euler equation corresponds to the first stage of the consumer's optimization problem.

The problem with this approach is that (at least without adding error terms) the demand system corresponding to the first step consists of deterministic relations. This implies that, if such a model is used in an empirical application, these relations should hold exactly for each observation in the data set. As this will generally not be the case, the demand system is usually amended by simply tacking on error terms to the demand equations. Some examples of this approach can be found in Blundell (1987), Blundell, Browning and Meghir (1988), and Alessie, Kapteyn and Melenberg (1989).

The main drawback to this approach is its ad hoc character: one adds error terms to the demand system of the second stage (of the consumer's optimization problem), without taking account of the implications of this additional stochastic structure for the first stage. Hence, it may be possible that the assumptions with respect to these ad hoc error terms are incompatible with the Euler equation corresponding to the first stage. For example, the probability distribution of the consumption goods induced by the additional error terms may conflict with the moment restrictions resulting from the Euler equations.

To give the imposition of the additional error structure a sound theoretical basis, one should incorporate the additional error structure from the outset, i.e., include it in the life cycle model before applying the two stage budgeting framework. However, to enable estimation, it is usually required that the error terms not only appear additively in the demand system, but also do not affect the first stage. It may not be easy to incorporate the additional error terms such that this requirement is met.

The problems with the estimation of the second stage mentioned so far could induce one to use only the first stage equation for estimating the parameters of interest, and ignore the intratemporal relationships altogether. However, only using the Euler equation corresponding to the first stage is often insufficient for obtaining estimates of all parameters of interest. A possible way out of this problem could be to use the multi-good life cycle consumption model not in the two stage budgeting format, but in its original formulation. By using this representation one can derive a system of Euler equations (for instance, one for each good). It is more likely that one can estimate all parameters of interest from such a system, than if one uses only the single Euler equation corresponding to the first stage.

A shortcoming which both of these approaches have in common, is that generally the resulting estimates will not satisfy the corresponding intratemporal relations, still indicating model-misspecification. Finally, trying to estimate the intertemporal relation(s) taking into account the corresponding intratemporal identities by imposing them as restrictions on the parameters usually is also not a feasible approach, as the intratemporal identities often imply conflicting restrictions on the parameters.[1]

It is, therefore, concluded that extending Hall's model to deal with consumption decisions concerning disaggregated consumption instead of total consumption introduces some problems which make a further modification of the model necessary. The remainder of this section is devoted to a discussion of some possibilities which have been suggested in the literature.

Well-known approaches to avoid intratemporal deterministic relations are the random preference approach, which is based on the assumption that the researcher does not exactly know the functional form of the utility function, and the measurement errors approach. An example of the first approach is MaCurdy (1983). An example in which measurement errors are included in the life cycle framework is Altonji and Siow (1987).

The strength of the Hall (1978) approach is that by making only relatively few assumptions one is nevertheless able to obtain, using the first order conditions, equations on the basis of which estimation and testing are straightforward, even if one chooses quite general forms of the life cycle model. A main disadvantage of the two approaches mentioned is their limited applicability.[2] Only if one is prepared to consider restricted formulations of the life cycle model, and limits one's interest to particular specifications of the utility function, one is (generally) able to obtain equations which make estimation and testing of the life cycle model possible.[3] These remarks especially apply if one wishes to take into account additional binding restrictions, such as nonnegativity constraints. In such cases one has to deal with extra Lagrange multipliers. Usually additional, often restrictive, assumptions are needed to handle these multipliers in a satisfactory way.[4]

The foregoing indicates that these two approaches in general undo some of the advantages of the Euler equation approach. Therefore, instead of using either one of them, a third possibility, suggested by Melenberg and Alessie (1989), is considered. These authors proposed to avoid deterministic intratemporal relations by introducing intratemporal uncertainty. In life cycle models consumers are usually supposed to make their decisions at the beginning of the observation periods, where the observation periods are determined by the data set at hand. Subsequently, the assumption is imposed that all the uncertainty inducing variables of a particular observation period are known by the consumer at the beginning of that observation period. This means that there is, in fact, only intertemporal uncertainty, i.e., only variables which realisations will occur in future periods are supposed to induce uncertainty. Put differently, there is only intertemporal planning: the consumption quantities of the present period are chosen deterministically, future consumption bundles are planned.

However, it is very well possible that a consumer, in deciding upon consumption at the beginning of a particular period, does not yet know the outcomes of all the random variables in that period. If this is the case, one should not only allow for intertemporal planning in life cycle models, but also for intratemporal planning: in a particular period not all the components of the vector of consumption quantities, corresponding to that same period, need to be chosen deterministically; some of the components may be planned.[5]

This can be illustrated by means of the following example. Assume that prices are not yet known at the beginning of the observation period, but that they become known during that period. In this case a consumer should wait until all prices have been realized in order to be able to choose the quantities of that period deterministically. But this actually means that a consumer decides upon consumption at the end of the period. This may be considered to be somewhat unrealistic. Instead, the case is considered in which a consumer still decides upon consumption at the beginning of a period, using the information which is then available. In order to be able to take into account the dependence upon the prices, whose realisations are not yet known, the consumption decisions of this period must now be in terms of a plan: the consumer decides upon the optimal consumption *functions*, where the arguments of these functions are the prices, and the images are the consumption quantities. Once prices are known, consumption quantities are known.

This approach can be used to modify the standard multi-good life cycle model in such a way that the intratemporal deterministic relations do not show up. This can be illustrated by the earlier introduced example dealing with prices. The point is that it is not necessary to assume that each particular component of the consumption vector of a particular period depends upon exactly the same prices (corresponding to that period). Different components may depend upon different prices. This situation will occur, for instance, if the realisation of prices takes place in some order.[6] In this case one can assume that the good corresponding to the price whose realisation occurs first only depends upon that price; the good corresponding to the price whose realisation occurs next depends upon its own price and the price of the first good, and so on. The good corresponding to the price whose realisation is the last one depends upon all prices.[7]

With such a modification, which need not necessarily be in terms of prices, but may also be in terms of other uncertainty inducing variables, the deterministic identities do not show up in general. Instead one can derive intratemporal stochastic relations, similar to the Euler equations. Subsequently, it seems natural to base the estimation on the resulting system of inter- and intratemporal moments characterising the optimum.

Notice that the advantage of this approach, compared to the approaches discussed before, is that it does not require the imposition of additional assumptions, but makes use of the already existing structure of the life cycle model. As a consequence, one is not restricted in the choice of the model formulation and, in particular, one is not restricted in the specification of the functional forms. Moreover, this modification includes Hall's standard multi-good version as a special case. The formal way the intratemporal uncertainty can be incorporated in the multi-good version of Hall's (1978) life cycle model can be found in Melenberg and Alessie (1989). In section 3 a more intuitive argument is given.

In section 4, this approach will be applied to estimate two types of life cycle consumption models. The first one is a basic version with two goods: food and non-food. The second type of life cycle consumption model also deals with two goods, but now vacation and non-vacation. Monthly expenditures on vacation are often equal to zero, so this case can serve as an example of a model in which the non-negativity constraints become important.

3. First Order Conditions and Moments.

3.1 The Model.

In this subsection the consequences of including intra-temporal uncertainty in the life cycle consumption model are considered, using a multi-good version of Hall's framework. In the next subsection the derivation of Euler equations and, more generally, of moment restrictions that can be used in estimating and testing two versions of the life cycle model are presented.

The life cycle models that are considered are those in which a consumer is only confronted with so-called exogenous uncertainty, induced by variables like income, prices, interest rates, and taste shifters. By uncertainty is meant that the values of the variables concerned are not known to the consumer at the moment the consumer determines the consumption for (the remainder of) the lifetime, but the probability distribution of these variables is known; by exogeneity is meant that this probability distribution cannot be influenced by the consumption decisions.[8] The uncertainty inducing variables will be called input variables.

In the standard approach, cf. Hall (1978), it is assumed that in a particular period t the consumer knows the realizations of the input variables up to and including period t, whereas the variables dated t+1 or later are uncertain. In period t the consumer is supposed to determine period t's consumption and to plan consumption for the periods t+1, t+2,...,L, with L the consumer's lifetime. The planned consumption of period τ, $\tau > t$, is allowed to depend upon the input variables up to and including period τ.

From a mathematical point of view this means that the consumer's choice set for period t is assumed to consist of (consumption) functions of the input variables. The functions indexed t (corresponding to consumption in period t) are deterministic, and the functions indexed τ, $\tau>t$, corresponding to (planned) consumption of period τ, are functions of all input variables up to and including period τ. As a consequence, planned consumption of period $\tau > t$, is a random variable, where the randomness is induced by the input variables in periods $t+1,\dots,\tau$. Of course, the consumption functions also should satisfy additional restrictions as implied by, e.g., the lifetime wealth budget constraint. The consumer chooses from the resulting choice set a vector of consumption functions by maximizing some objective function like the von Neumann-Morgenstern expected utility function.

This approach leads to a model of the following type which is called the <u>standard life cycle model</u>, cf., for example, Hall (1978) for the one good version of this model. A consumer solves the following problems during his or her lifetime for $t=1,\dots,L$, consecutively, where the maximization is with respect to $(q_t',\dots,q_L')'$,

$$\begin{aligned} &\text{Max } E_t\textstyle\sum_{\tau=t}^{L} u_\tau(q_\tau) \\ &\text{s.t. } \textstyle\sum_{\tau=t}^{L} i_{t\tau}p_\tau' q_\tau \leq (1+r_{t-1})A_{t-1} + \textstyle\sum_{\tau=t}^{L} i_{t\tau}y_\tau \end{aligned} \tag{3.1.1}$$

where

$q_\tau = (q_{1,\tau},\dots,q_{M,\tau})'$: M-dimensional vector of quantities of goods in period τ, $\tau=t,\dots,L$,

$p_\tau = (p_{1,\tau},\dots,p_{M,\tau})'$: M-dimensional price vector of the goods in period τ, $\tau=t,\dots,L$,

y_τ : Nominal non-property income in period τ, $\tau=t,\dots,L$,

r_τ : Nominal interest rate in period τ, $\tau=t-1,\dots,L$,

$i_{tt} = 1$,

$i_{t\tau} = \prod_{j=t}^{\tau-1} (1+r_j)^{-1}$, $\tau=t+1,\dots,L$,

A_{t-1}: Non-human wealth at the end of period t-1,

$E_t\sum_{\tau=t}^{L} u_\tau(.)$: Expected utility function, conditional upon all information *up to and including period t.*

Notice that once q_t is chosen in period t the amount of non-human wealth at the end of that period is given by

$$A_t = (1+r_{t-1})A_{t-1} + y_t - p_t'q_t.$$

Suppose that in this model prices, interest rates, and income are the input variables. Then, in period t, the decision variables concerning period τ, $\tau > t$, are allowed to depend upon (at least) the input variables unknown in period t contained in the set[9]

$$\{y_{t+1},p_{t+1},r_t,\dots,y_\tau,p_\tau,r_{\tau-1}\}. \tag{3.1.2}$$

The expectation operator E_t is conditional upon the variables contained in the information set denoted by I_t, which is assumed to include at least the set $\{y_1,p_1,r_0,\dots,y_t,p_t,r_{t-1}\}$. Hence one can write, for some function f(.):

$$E_t[f(.)] = E[f(.)|I_t].$$

Hall (1978) only considered total consumption, and obtained the corresponding Euler equation by means of a calculus of variations technique. In the multi-good case studied here, that same technique can be applied to obtain not only a system of Euler equations, but also intratemporal relations between marginal utilities. To demonstrate this, choose as variations

$$\begin{aligned} &q_{kt} + \varepsilon/p_{kt} \\ & \qquad\qquad\qquad k,\ell \in \{1,..,M\} \quad k \neq \ell \\ &q_{\ell t} - \varepsilon/p_{\ell t}. \end{aligned}$$

Substituting these variations into model (3.1.1) and assuming that the life time wealth budget constraint is binding, results, after differentiation with respect to ε and evaluating the derivative in $\varepsilon=0$, in

$$(1/p_{kt})(\partial u_t(q_t)/\partial q_{kt}) = (1/p_{\ell t})(\partial u_t(q_t)/\partial q_{\ell t}). \tag{3.1.3}$$

These intratemporal relations, consistent with model (3.1.1), are *deterministic*.

As was argued in section 2, it is unlikely that these relations will be satisfied exactly by the data. Melenberg and Alessie (1989), therefore, suggested a modification of the standard life cycle model that avoids the presence of such deterministic relationships. From a technical point of view, their approach basically boils down to use, in case of period τ, not just one set of input variables upon which *all* $q_{1\tau},\ldots,q_{M\tau}$ are assumed to depend. Instead they allow for M different sets in each period τ, one for each consumption good $q_{m\tau}$, $m=1,\ldots,M$. To be precise, define -still assuming that only prices, income and interest rates induce uncertainty- for each $\tau \geq t$: $\eta_\tau = (y_\tau,p_\tau',r_{\tau-1})'$. Then it is assumed that $\eta_\tau = (\underline{\eta}_\tau',\bar{\eta}_\tau')'$, with the interpretation that the realization of $\underline{\eta}_\tau$ is known at the beginning of period τ, whereas the realization of $\bar{\eta}_\tau$ is not yet known at the beginning of period τ. Using this notation, the set of input variables corresponding to good m in period τ is no longer given by (3.1.2), but becomes

$$\{\bar{\eta}_t,\eta_{t+1},\ldots,\eta_{\tau-1},\underline{\eta}_\tau,\eta_{m\tau}\},$$

where $\eta_{m\tau}$ consists of those elements of $\bar{\eta}_\tau$, which the consumer knows when deciding upon $q_{m\tau}$. Compared to the standard formulation there are two modifications regarding the set of input variables on which $q_{m\tau}$ is allowed to depend: the first one is that η_τ together with $\eta_{m\tau}$ replaces $(y_\tau, p'_\tau, r_{\tau-1})'$; the second one is that $\bar{\eta}_t$ is added. By assuming that $\eta_{m\tau}$ varies with m, one obtains different sets of input variables corresponding to the different goods. This modification implies that the expectation operator E_t now becomes conditional upon the variables contained in the original information set I_t, except the variables of period t which realisations are not yet known at the beginning of period t, the moment at which the consumer is supposed to decide in period t. Thus $\bar{\eta}_t$ is exluded from I_t. Denote the new information set by I'_t. Then one can write

$$E_t[f(.)] = E[f(.)|I'_t].$$

The same symbol E_t is used, since this symbol just reflects taking conditional expectations at the beginning of period t, which is not changed in the present approach. What does change, is the information set available at that time.

Applying a general Lagrange multiplier rule, as, for instance, given in Neustadt (1976, ch. III), Melenberg and Alessie (1989) show[10] that with these modifications, the first order conditions of the life cycle consumption model (3.1.1) become of the following form. There should hold for *all* possible functions $(h'_t,\ldots,h'_L)'$ of the input variables, where $h_\tau=(h_{1\tau},\ldots,h_{M\tau})'$, $\tau=t,\ldots,L$, and where $h_{m\tau}$ is allowed to depend upon the same input variables as $q_{m\tau}$, $m=1,\ldots,M$, $\tau=t,\ldots,L$,

$$E_t\ [\Sigma^L_{\tau=t}Du_\tau(q_\tau)'h_\tau - \ell_t.\Sigma^L_{\tau=t}i_{t\tau}p'_\tau h_\tau] = 0. \tag{3.1.4}$$

Here $Du_\tau(q_\tau)$ denotes the vector of partial derivatives of $u_\tau(.)$ evaluated at the point q_τ and ℓ_t is the Lagrange multiplier corresponding to the budget constraint. The Lagrange multiplier is a function of *all* input variables.

It is now possible to illustrate that by allowing for intratemporal uncertainty the intratemporal relations need no longer be deterministic. Consider as an example the ordering in the consumption of different goods, which was given in section 2 as a possible explanation for the presence of intratemporal uncertainty and which is maintained in the empirical application of section 4. Suppose that $\eta_{k\tau}$ includes $p_{k\tau}$, but does not include $p_{\ell\tau}$, whereas $\eta_{\ell\tau}$ contains both $p_{k\tau}$ and $p_{\ell\tau}$. If one now uses in (3.1.4) choices for the $h_{i\tau}$ that correspond to the variations used in in the derivation of (3.1.3), i.e., if one substitutes

$$\begin{aligned} h_{kt} &= +\,(1/p_{kt}) \\ h_{\ell t} &= -\,(1/p_{\ell t}) \end{aligned} \tag{3.1.5}$$

into (3.1.4), and set all other $h_{i\tau}$ equal to zero, the deterministic intratemporal relationships (3.1.3) do not show up. Instead, substituting (3.1.5) results in

$$E_t[(1/p_{kt})(\partial u_t(q_t)/\partial q_{kt}) - (1/p_{\ell t})(\partial u_t(q_t)/\partial q_{\ell t})] = 0. \tag{3.1.6}$$

Notice that in this modified model, the conditional expectation operator is still needed since $p_{\ell t}$ has to be averaged out.

3.2. The construction of moments.

As demonstrated above in equations (3.1.4)-(3.1.6), the construction of moments becomes rather straighforward once the first order conditions have been formulated. The derivation and formulation of the first order conditions itself is more technical and can be found in Melenberg and Alessie (1989), who apply Neustadt (1976, ch. III). Their framework is used here in order to derive moment restrictions for two versions of the life cycle consumption model with intratemporal uncertainty. The first one is the basic version with only a lifetime wealth budget constraint given in (3.1.1). The second version consists of this model with two goods,[11] say, $q_{1\tau}$ and $q_{2\tau}$, extended with additional inequality constraints with respect to the second good

$$q_{2\tau} \geq 0, \ \tau=t,\ldots,L. \tag{3.2.1}$$

The addition of these inequality constraints is, of course, only meaningful if they are binding for a nonzero fraction of sample observations. This second version is an example of the type of models discussed in section 2, i.e., models that, in order to enable estimation and testing, usually require additional assumptions in the absence of intratemporal uncertainty.

First, the construction of moments for model (3.1.1) is discussed, after which the derivation of moments for model (3.1.1) if the inequality constraints (3.2.1) are included is presented.

i) The basic life cycle consumption model.

Using (3.1.4) the Euler equations (in terms of observables only) can easily be obtained. To illustrate this consider the derivation of an Euler equation with respect to good m, $m \in \{1,...,M\}$, relating periods t and t+1. Choose

$$h_{mt} = -1,\ h_{m,t+1} = p_{mt}/(i_{t,t+1}p_{m,t+1}) \tag{3.2.2}$$

and choose all other $h_{i\tau}$ equal to zero. Substituting these choices into equation (3.1.4) immediately results in[12]

$$\begin{aligned} E_t[(p_{mt}/(i_{t,t+1}p_{m,t+1}))\ \partial u_{t+1}(q_{t+1})/\partial q_{m,t+1} - \\ \partial u_t(q_t)/\partial q_{mt}] = 0. \end{aligned} \tag{3.2.3}$$

Unconditional moments, which should equal zero and which make estimation and testing possible, follow from (3.2.3). They take the form

$$E\{[Diag_{t,t+1}Du_{t+1}(q_{t+1}) - Du_t(q_t)]\otimes z_t\}=0, \tag{3.2.4}$$

with

$$Diag_{t,t+1}=Diag([p_{1t}/(i_{t,t+1}p_{1,t+1})],...,[p_{Mt}/(i_{t,t+1}p_{M,t+1})]),$$

and where $z_t = (z_{1t},...,z_{Kt})'$ is any function taking values in $\mathbb{R}^K$ (K some positive integer) which only depends on what is known by the consumer at the beginning of planning period t (i.e., the information set I'_t).

The above equations concern the *intertemporal* relationship between marginal utilities. As was already shown in (3.1.6) for period t, it is also posible to obtain *intratemporal* relationships. By taking $h_{\tau'} = 0$, $\tau' \neq t$, $h_{jt} = 0$, $j \neq k,\ell$, one obtains, for example,

$$E_t[\partial u_t(q_t)/\partial q_{kt}.h_{kt}+\partial u_t(q_t)/\partial q_{\ell t}.h_{\ell t}-\ell_t(p_{kt}h_{kt}+p_{\ell t}h_{\ell t})]=0, \tag{3.2.5}$$

where the extra indices k and ℓ refer to good k and good ℓ, respectively. By following the same procedure as was used in case of the Euler equation it easily follows that

$$E_t[(\partial u_t(q_t)/\partial q_{kt})/p_{kt} - (\partial u_t(q_t)/\partial q_{\ell t})/p_{\ell t}] = 0, \tag{3.2.6}$$

and similarly to (3.2.4) one can construct unconditional moments.

ii) Additional inequality constraints.

From Melenberg and Alessie (1989) (cf. also Neustadt (1976, ch III)) one can derive that, for $(q_t',\ldots,q_L')'$ to be optimal in case of model (3.1.1) with two goods, and extended with the inequality constraints (3.2.1), there should hold for *all* $(h_t',\ldots,h_L')'$ similarly defined as in the case of the standard life cycle model,

$$\begin{aligned}&E_t[\textstyle\sum_{\tau=t}^{L}(\partial u_\tau/\partial q_{1\tau})h_{1\tau}+(\partial u_\tau/\partial q_{2\tau})h_{2\tau}-\ell_t\sum_{\tau=t}^{L}i_{t\tau}p_\tau'h_\tau+\\&\quad\textstyle\sum_{\tau=t}^{L}\mu_{2\tau}h_{2\tau}] = 0,\end{aligned} \tag{3.2.7}$$

such that

$$E_t[q_{2\tau}\ \mu_{2\tau}] = 0,\ \tau=t,\ldots,L, \tag{3.2.8}$$

where $\mu_{2\tau}$, $\tau=t,\ldots,L$, the (generalized) Lagrange multipliers corresponding to the nonnegativity constraints, are nonnegative. These additional Lagrange multipliers are allowed to depend upon the same input variables as $q_{2\tau}$, $\tau=t,\ldots,L$.

To obtain a first moment in terms of observables only, choose

$$h_{1t} = -1/p_{1t},\ h_{1,t+1} = 1/(i_{t,t+1}p_{1,t+1}),$$

and the other $h_{i\tau}$ equal to zero. The resulting Euler equation relating $\partial u_t/\partial q_{1t}$ and $\partial u_{t+1}/\partial q_{1,t+1}$ becomes:

$$E_t[[1/(i_{t,t+1}p_{1,t+1})]\ \partial u_{t+1}(q_{t+1})/\partial q_{1,t+1} - [1/p_{1t}]\ \partial u_t(q_t)/\partial q_{1t}] = 0. \tag{3.2.9}$$

A second moment can be derived by choosing[13]

$$h_{2t} = (-1/p_{2t})I_{(0,\infty)}(q_{2t}),$$

$$h_{1,t+1} = (1/(i_{t,t+1}p_{1,t+1}))I_{(0,\infty)}(q_{2t}),$$

with $I_{(0,\infty)}(q_{2t})$ the usual indicator function, resulting in

$$E_t[[(1/(i_{t,t+1}p_{1,t+1}))\partial u_{t+1}(q_{t+1})/\partial q_{1,t+1} - (1/p_{2t})\partial u_t(q_t)/\partial q_{2,t}]I_{(0,\infty)}(q_{2t})] = 0. \tag{3.2.10}$$

Notice that (3.2.8) together with the nonnegativity of μ_{2t} ensure that $E_t[\mu_{2t}h_{2t}] = 0$, for this particular choice of h_{2t}. Notice, in addition, that one could, for example, also have used q_{2t} instead of $I_{(0;\infty)}(q_{2t})$ in the construction of h_{2t} and $h_{1,t+1}$.

Compared with the basic model, the intertemporal Euler equation regarding good 2 and the intratemporal moment concerning period t are replaced by the moment (3.2.10) in order to eliminate the (unknown) multipliers corresponding to the non-negativity constraints. Notice, moreover, that when using one of the alternative approaches for incorporating additional uncertainty into the model discussed in section 2, one is generally also able to derive a system of moments similar to (3.2.7). However, the construction needed to eliminate the unknown multipliers in order to obtain an equation similar to (3.2.10), i.e., in observables only, usually will require additional assumptions, not needed in the present approach. Finally, observe that the systems derived here are just some possible combinations of the first order conditions. Other combinations can also be derived.

4. Empirical Application.

4.1 The Data.

The objective of this section is to assess the empirical relevance of the life cycle model with intratemporal uncertainty. This will be done on the basis of the two-goods version of both the basic model and a model with additional non-negativity constraints. For both models, two specifications will be estimated.

The data come from the so-called 'Intomart consumer expenditure panel'. This panel contains information on monthly expenditures of households on several commodity categories, and a number of demographic characteristics of these households (including social class and household composition) which are registered on an annual basis. Notice that the data refer to households, whereas the models discussed thusfar are concerned with individual consumers. In order to be able to use these data, one must assume that decisions are made at the household level, not at the individual level. This often made assumption will be maintained throughout this study. As prices were added the national price indices corresponding to the commodity classes as reported by the Netherlands Central Bureau of Statistics. The panel covers the forty-two months from April 1984 through September 1987.

There are some characteristics of the data set that need to be reported. Firstly, almost no household participates in the panel for the complete spell April 1984-September 1987. Only 91 of the 2,897 households participate in all 42 periods.[14] Secondly, when constructing sample analogues of the moments that are used in estimation, different moments correspond with different data requirements. The way in which the moment restrictions are formulated (see subsection 4.2), implies that all 32,456 observations (households times periods) can be used for constructing sample analogues of the intratemporal moments which have a demographic variable as instrument. For the intratemporal relations which have the one period lagged expenditures or price as instrument, as well as for the intertemporal ones which have a demographic variable as instrument, only those households participating at least two consecutive periods are used.[15] This requirement is met by 29,732 observations reported by 2,566 households. Finally, for the intertemporal restrictions which have the one period lagged expenditures or price as instrument, only those households are used which participate at least three consecutive periods. This requirement reduces the number of observations that can be used to 27,334, which are reported by 2,382 households. It is assumed that both types of selection (attrition in the original panel and selection resulting from creating sample analogues of the different moment restrictions) are random.

Finally, a remark needs to be made concerning the nature of the data. The panel used for estimation consists of observations on the expenditures of households, whereas the model to be estimated is formulated in terms of consumption. Given the short measurement period (a month), there may exist a difference between these two quantities which may even be considerable. In this chapter the (often made) assumption that consumption and expenditures are equal will be maintained. The consequences of taking account of the difference between consumption and expenditures will be taken up in subsequent chapters, especially chapter 4.

4.2 Derivation of Moments.

As mentioned before, the application is limited to the two-goods case. The categories considered are food and non-food for the application of the basic life cycle model (3.1.1), and vacation and non-vacation for the application of the model which includes the non-negativity constraints (3.2.1). Depending on which model is estimated, either food or vacation is the second good.

As can be seen from Table 1, vacation is a clear example of an infrequently purchased good, which implies that the non-negativity constraint for this good will be binding for many observations. Hence, the corresponding application can serve as an example of the sort of models which often require additional distributional assumptions to enable estimation, if one of the other approaches discussed in section 2 (i.e. including random preferences or measurement errors) is employed to make the intratemporal equations non-deterministic.

Table 1. Percentage of households with zero vacation expenditures[1)]

Period	NH	PZ	Period	NH	PZ	Period	NH	PZ
1	921	79.8	15	753	71.2	29	798	69.3
2	966	74.1	16	757	63.0	30	787	80.8
3	884	66.6	17	767	71.0	31	837	83.2
4	922	59.2	18	789	80.0	32	858	90.2
5	855	68.3	19	806	86.0	33	978	89.3
6	757	81.5	20	764	91.4	34	956	84.1
7	889	85.9	21	742	90.2	35	1022	83.5
8	849	91.5	22	676	84.2	36	1018	80.6
9	789	89.2	23	667	83.2	37	981	78.5
10	736	85.7	24	680	82.7	38	1024	71.5
11	693	82.1	25	706	78.1	39	1052	66.5
12	856	82.9	26	676	71.0	40	968	60.6
13	816	77.6	27	776	69.9	41	954	66.4
14	751	71.7	28	818	59.8	42	898	76.8

1) NH = number of households participating in the original panel in a certain month

PZ = percentage of these households that register zero expenditures for vacation in that month

period 1 = April 1984

period 42 = September 1987

In the empirical applications that are considered in this section, the following specification is chosen. The intratemporal utility function is assumed to depend on τ only through the discounting factor, i.e. $u_\tau(.) = (\frac{1}{1+\rho})^{\tau-t}u(.)$ with ρ the time preference parameter, which is assumed to be constant over time as well as over households. Secondly, as it is not clear which observable interest rate corresponds to the interest rate of the model, the r_τ in (3.1.1) are taken to be unknown parameters. Similar to Hall (1978), the interest rate is assumed to remain constant over time, an assumption which reduces the number of parameters considerably, but implies that it is not possible to estimate the time preference parameter ρ. Only the quotient $(1+r)/(1+\rho)$ can be estimated. For the intratemporal utility function u(.) the following quadratic specification is chosen, where the normalization $a \cdot c-b^2=1$ is imposed to ensure identification[16]:

$$u(q_{h,\tau}) = \frac{1}{2}\{a \cdot q_{h,\tau,1}^2+2 \cdot b \cdot q_{h,\tau,1} \cdot q_{h,\tau,2}+c \cdot q_{h,\tau,2}^2\} +$$

$$d \cdot q_{h,\tau,1}+e \cdot q_{h,\tau,2}, \qquad (4.2.1)$$

where a $(= (1+b^2)/c)$, b, c, d and e are parameters to be estimated.

As a generalization of this basic version the parameters d and e will be made household dependent, thus allowing the bliss point[17] of the quadratic utility function to be household specific. The particular form in which this is modelled, is by letting these parameters depend on the logarithm of the household size:

$$d_h = d_0+d_1 \cdot \log(fs_h), \qquad (4.2.2)$$
$$e_h = e_0+e_1 \cdot \log(fs_h), \qquad (4.2.3)$$

were fs_h is the household size of household h.

As far as the intratemporal uncertainty is concerned, the assumption made in the example which was given in section 2 is maintained: the presence of intratemporal uncertainty results from the fact that goods are bought in some order during a period. No information regarding this ordering is needed to enable estimation. For example, it may (and probably will) vary in some unknown way over households and over time, but this does not hamper estimation in any way.

When applying the approach suggested by Melenberg and Alessie (1989), the moment restrictions derived in section 3 are used. Let $z_{h,t}^i$ i=1,..,5, be vector-valued functions of variables known by consumer h at the beginning of period t, t=1,..,L. For the food/non-food case, a system of moment restrictions can easily be derived from the equations (3.2.4) and (3.2.6). One such system, making use of the quadratic utility function (4.2.1), is the following one (the formulation of the moments for an arbitrary utility function $u(\cdot)$ is given in appendix B):

-intratemporal:

$$E\ [\{[(1+b^2)/c]\cdot\frac{q_{h,t,1}}{p_{t,1}} + b\cdot[\frac{q_{h,t,2}}{p_{t,1}} - \frac{q_{h,t,1}}{p_{t,2}}] - c\cdot\frac{q_{h,t,2}}{p_{t,2}} + \frac{d}{p_{t,1}} - \frac{e}{p_{t,2}}\}z^1_{h,t}] = 0, \qquad (4.2.4)$$

for t=1,...,41;

-intertemporal:

$$E\ [\{[(1+b^2)/c]\cdot[\frac{q_{h,t,1}}{p_{t,1}} - (\frac{1+r}{1+\rho})\cdot\frac{q_{h,t+1,1}}{p_{t+1,1}}] + b\cdot[\frac{q_{h,t,2}}{p_{t,1}} - (\frac{1+r}{1+\rho})\cdot\frac{q_{h,t+1,2}}{p_{t+1,1}}] + d\cdot[\frac{1}{p_{t,1}} - (\frac{1+r}{1+\rho})\frac{1}{p_{t+1,1}}]\}\ z^2_{h,t}] = 0, \qquad (4.2.5)$$

for t=1,...,41;

$$E\ [\{[(1+b^2)/c]\cdot\frac{q_{h,41,1}}{p_{41,1}} + b\cdot[\frac{q_{h,41,2}}{p_{41,1}} - (\frac{1+r}{1+\rho})\cdot\frac{q_{h,42,1}}{p_{42,2}}] - c\cdot(\frac{1+r}{1+\rho})\cdot\frac{q_{h,42,2}}{p_{42,2}} + \frac{d}{p_{41,1}} - (\frac{1+r}{1+\rho})\cdot\frac{e}{p_{42,2}}\}z^3_{h,41}] = 0. \qquad (4.2.6)$$

Notice that, in contrast with equation (4.2.5), equation (4.2.6) represents the Euler equation linking the expected marginal utility of the two different goods. Instead of this intertemporal equation, one can also use the intratemporal equation for the last period. This latter possibility will, because of the averaging over time of the moments (see below), result in fewer moment restrictions, and in fewer degrees of freedom. The effect of the choice of last period's moment on the estimation results, will be investigated in the next subsection.

For the vacation/non-vacation case one could apply the moments given by (3.2.9) and (3.2.10). A disadvantage of (3.2.10) is that it only uses those households in period t, which register a positive amount of consumption of vacation in this period. As can be seen from Table 1, this implies that for this second moment, most observations will be left unused in estimation. Although from a theoretical point of view not using these observations must not affect the outcome, it turned out to lead to some numerical problems in the empirical application.[18] Therefore, the first order conditions were combined in a different way, in order to derive a moment restriction which does not suffer from this drawback. If the household does not report holiday expenditures in period t, just the Euler equation for the non-vacation good results; otherwise the Euler equation linking the expected marginal utility of period t's consumption of holidays with the expected marginal utility of consumption of the other good in period t+1 is added. The two resulting (unconditional) moments which are used for estimation are the following ones (where the general formulation of these moments can again be found in appendix B):

$$E\ [\{[(1+b^2)/c]\cdot[\frac{q_{h,t,1}}{p_{t,1}} - (\frac{1+r}{1+\rho})\cdot\frac{q_{h,t+1,1}}{p_{t+1,1}}] +$$

$$b\cdot[\frac{q_{h,t,2}}{p_{t,1}} - (\frac{1+r}{1+\rho})\cdot\frac{q_{h,t+1,2}}{p_{t+1,1}}] +$$

$$d\cdot[\frac{1}{p_{t,1}} - (\frac{1+r}{1+\rho})\cdot\frac{1}{p_{t+1,1}}]\}\ z^4_{h,t}] = 0, \qquad (4.2.7)$$

$$E\ [\{[(1+b^2)/c]\cdot q_{h,t,1} + b\cdot q_{h,t,2} + d -$$

$$(b\cdot q_{h,t,1} + c\cdot q_{h,t,2} + e)\cdot I_{(0,\infty)}(q_{h,t,2})) -$$

$$(\frac{1+r}{1+\rho})\cdot\ ([(1+b^2)/c]\cdot q_{h,t+1,1} + b\cdot q_{h,t+1,2} + d)\cdot$$

$$(\frac{p_{t,1} - p_{t,2}\cdot I_{(0,\infty)}(q_{h,t,2})}{p_{t+1,1}})]\}\ z^5_{h,t}] = 0, \qquad (4.2.8)$$

for t=1,...,41.

Notice that equation (4.2.7) is the same as equation (4.2.5), whereas equation (4.2.8) is a linear transformation of equation (4.2.7), extended with the aforementioned Euler equation which links the expected marginal utility of the two goods in period t and t+1, respectively.

When constructing sample analogues of the two systems of moment restrictions presented above, it is often observed that one should be aware of possible effects of economy-wide shocks. As pointed out by, for instance, Chamberlain (1984), Hayashi (1985a) and Hotz, Kydland and Sedlacek (1988), if such shocks are present, averaging over time is essential to ensure the consistency of the estimators. Therefore, the systems of moment restrictions as given in (4.2.4)-(4.2.8) was not estimated, but these relations were first averaged over time.[19] Notice that equation (4.2.6) only concerns period 41. Hence, if an economy-wide shock is present in this period, it may result in inconsistent estimates. To take account of this potential problem, the food versions in which equation (4.2.6) is replaced by the intratemporal equation of period 42 were also estimated. Since this latter equation is included in the averaging of the intratemporal moments, it is possible that an economy-wide shock in this equation can, loosely speaking, be compensated by an economy-wide shock in another period.

Although different sets of instruments can be used to estimate the different moment restrictions, all are estimated using the same set of instruments. It consists of the set of demographic variables described in appendix A, extended with the one period lagged food expenditures and price of food for the basic version, and with the one period lagged vacation expenditures for the extended version.[20] The resulting systems of moment restictions are estimated by means of the Generalized Method of Moments (GMM) (using the efficient weighting matrices) as discussed in, for instance, Hansen and Singleton (1982). In the next subsection, the estimation results of two versions of the quadratic utility function are presented, for both systems of moment restrictions introduced in this subsection.

4.3 Estimation results.

In this subsection the estimation results of the various cases, specified in the previous subsection, are presented. Moreover, some specification tests are performed.

In Table 2 the estimation results for both versions of the two models are given. A number of observations can be made from this table.

The first one is that choosing either an intertemporal (food1b and food2b) or an intratemporal (food1a and food2a) moment for the last period has only limited consequences for the estimates (compare versions a and b of the food case).

Comparing the food and holiday cases one can see a clear difference which does not so much concern the estimates, but the corresponding standard errors. Especially the estimates of the parameters corresponding to the linear part of the utility function, i.e., d_0, d_1, e_0 and e_1, have large standard errors in the food cases. A possible explanation for this is that, as can be seen from the moment restrictions given in subsection 4.2, these parameters correspond with terms which are mainly determined by prices. Although all are rather stable during the survey period, the price variation in the food cases is even smaller than the variation in the holiday cases. Therefore, the estimates of these parameters are likely to be less precise in the food cases.

Turning next to the estimates themselves, it can be seen from Table 2 that the estimate of the parameter c is negative (and significant) for all cases, implying a strictly concave utility function, as required.[21]

Another condition that should hold for the models to be consistent with consumer theory, is that the bliss point (i.e., the top of the 'utility hill') is located such, that all observations are situated on the part of the utility function where it is increasing in both its arguments.[22] For the basic versions of the food case (food1a and food1b), this requirement is met by all reported food expenditures, and by all but 0.8% for version food1a and 0.9% for version food1b of the non-food expenditures. For the basic holiday version (holiday1), the percentage of wrongly situated observations rises to 2.8 for holiday and 2.1 for the non-holiday good, respectively.

The dependence of the parameters d and e on the logarithm of the household size for the household specific versions, implies a similar dependence for the bliss point. Hence, the aforementioned 'bliss point condition' must be checked for each household size separately. As can be seen from Table 2, the estimates of the parameters d_1 and e_1 are positive in all versions, implying that the bliss point increases with the household size, as one would expect. Notice that, although neither of the estimates of these parameters is significantly different from zero for the food versions, the values of the Wald statistic, T2, reported in Table 2, nevertheless indicate that they are jointly significant.

Checking the 'bliss point condition' for the household specific versions, it follows that for the food versions it is met, as far as food expenditures are concerned, by all observations except one for version a and except two for version b. For the non-food purchases, the percentage of violations varies somewhat with the household size (between 0% and 0.6%), but is around 0.2% for most household sizes. The percentages for the holiday case are somewhat larger, but do not differ in a dramatic way. The percentage of rejections for the holiday expenditures varies between 0 and 0.6, whereas this percentage lies between 0.4 and 2.4 for the non-holiday expenditures. All in all, the number of observations rejecting the 'bliss point condition' seems to be acceptable.

Furthermore, it can be seen from Table 2 that for all cases the term $(1+r)/(1+\rho)$ is estimated to be close to one. The small standard error for the household specific food cases implies that $(1+r)/(1+\rho)$ is significantly larger than one, which means that the time preference parameter ρ is smaller than the nominal interest rate. The corresponding estimates of $(1+r)/(1+\rho)$ indicate that this difference, although significant, is really quite small. Of greater importance is that under the assumption that r is positive, which does not seem too unrealistic since r is the nominal interest rate, these estimates imply for all four versions a positive value for the time preference parameter ρ. This contrasts with the negative estimates of ρ reported in the studies of Alessie, Melenberg and Kapteyn (1988), Hotz, Kydland and Sedlacek (1988) and Eichenbaum, Hansen and Singleton (1988). Since a negative value of ρ implies the postponement of all consumption until the last period, such an outcome is counterintuitive.

Finally, the results of Hansen and Singleton's (1982) test on overidentifying restrictions, which is a general misspecification test, are presented in Table 2. The resulting values for the food cases do not lead to rejection of the models. Moreover, they indicate that replacing intertemporal equation (4.2.6) by the intratemporal equation (4.2.4) for period 42 does not change the overall conclusion, but reduces the significance level considerably. Furthermore, comparing the basic food versions and the household specific food versions, shows that the household dependency that was introduced does not improve the test results, despite the earlier reported joint significance of the household effect. In contrast, for the holiday case, incorporating the household specific components in the utility function does lead to a considerable improvement, as it results in acceptance of the model.

The outcome that, for the food case, the value of the general misspecification test is larger for the extended model (i.e., the model with a household specific utility function) than for the basic model, can be explained by the fact that for each version the sample analogue of the optimal weighting matrix was used. Since they are constructed by taking the outer product of the sample analogues of the moments corresponding to a particular version (i.e. household specific moments for versions food2a and food2b), different versions have different weighting matrices. The test results indicate that the rather simple specifications that were estimated are, perhaps surprisingly, not rejected by the data.[23]

Table 2. Estimation results[1)]

Version	food1a	food1b	food2a	food2b	holiday1	holiday2
b	-0.118 (0.142)	-0.187 (0.145)	-0.103 (0.128)	-0.133 (0.131)	-0.771 (0.163)	-0.523 (0.182)
c	-1.639 (0.449)	-1.652 (0.464)	-2.579 (0.614)	-2.515 (0.631)	-1.856 (0.102)	-1.660 (0.069)
d_0	86.89 (44.85)	86.95 (69.13)	85.25 (96.37)	85.80 (240.6)	87.04 (24.30)	88.62 (28.11)
d_1	·	·	5.977 (97.46)	6.575 (227.4)	·	31.32 (13.57)
e_0	82.74 (43.62)	83.84 (67.30)	84.51 (93.54)	85.08 (234.0)	93.59 (24.42)	94.41 (28.61)
e_1	·	·	12.47 (94.42)	12.85 (221.2)	·	27.62 (14.04)
$\frac{1+r}{1+\rho}$	1.000 ($4\cdot10^{-4}$)	1.001 (0.001)	1.001 ($2\cdot10^{-4}$)	1.001 ($3\cdot10^{-4}$)	0.999 (0.009)	1.000 (0.003)
T1	21.1	15.7	30.9	24.3	31.5	18.8
df1	31	19	29	17	17	15
p1	0.909	0.677	0.370	0.112	0.017	0.222
T2	·	·	8.2	7.1	·	37.9
df2	·	·	2	2	·	2
p2	·	·	0.017	0.028	·	$6\cdot10^{-9}$

1) consumption measured in hundreds of guilders
standard errors in parentheses
food1a, food1b, holiday1 = basic version
food2a, food2b, holiday2 = version with household specific parameters d and e
food1a, food2a = version with intertemporal moment (4.2.6) for the last period
food1b, food2b = version with intratemporal moment (4.2.4) for the last period
T1 = chi-square value for Hansen and Singleton's misspecification test
df1 = degrees of freedom of misspecification test
p1 = significance level of misspecification test
T2 = value of Wald test on significance of combined household effect
df2 = degrees of freedom of Wald test
p2 = significance level of Wald test

5. Summary and conclusions.

In this paper a problem inherent in the often applied multi-good version of Hall's (1978) life cycle model was studied, i.e. the fact that the first order conditions characterizing the optimal consumption path do not only imply intertemporal Euler equations, but also deterministic intratemporal relations. As these deterministic relations will generally not hold exactly in empirical applications, their presence indicates a form of misspecification.

Several ways of modifying the life cycle model in order to overcome this problem were discussed. Because of its general applicability, the modification proposed by Melenberg and Alessie (1989) was chosen. They extend the standard life cycle model by dropping the assumption that there is no uncertainty within the consumer's decision period. Instead, the consumption plan for each period is allowed to depend on some input variables, which are still uncertain at the beginning of the period, but are realized during the period. As a consequence of the presence of this so-called intratemporal uncertainty, the intratemporal relations need no longer hold exactly for each separate consumer, but only 'on average', whilst the intertemporal Euler equations remain essentially unchanged.

In order to assess the empirical relevance of the modification, some two-good versions of the model were estimated, using a panel running for 42 periods during which 2,897 households participated, which resulted in a total of about 30,000 observations. The following conclusions can be drawn from the estimation results presented in section 4.

Firstly, the estimates are, by and large, in accordance with the theory, i.e., the estimated utility functions are concave and increasing in their arguments for almost all observations; the bliss points are increasing with household size; and in all versions the estimates imply a positive time preference parameter.

Secondly, the food versions indicate that using (the sample analogue of) a moment which is not averaged over time, has only a limited impact on the estimation results. Given that the observation period is a month, the absence of a substantial economy-wide shock is not surprising, since it may take some time before the effects of such a shock become apparent. The main influence is on the significance level of the general misspecification test. This effect is mainly the result of an increase in degrees of freedom, due to not including this moment in the averaging of the moments over time.

Furthermore, the results of Hansen and Singleton's (1982) misspecification test show that, apart from the basic holiday case, all estimated versions are accepted. Given the rather parsimonious specifications which were used, this result may be somewhat surprising.

Finally, when checking whether the intratemporal equations of the multi-good life cycle model hold exactly -the implicit assumption of the standard life cycle model- this turned out not to be the case for any version that was estimated. However, notice that this does not indicate the sort of additional randomness that should be incorporated into the standard model. For example, the moment restrictions corresponding to model (3.1.1) that were estimated in section 4, can also be obtained if one incorporates, instead of the intratemporal uncertainty, random preferences or measurement errors in the standard model.

Although the choice one makes regarding the source of the additional randomness will depend on the aim of the study, the general applicability of the intratemporal uncertainty framework can be an important advantage. By making use of this advantage, more complex life cycle models can also be estimated and tested. An example of such a model is the model which is considered in the next chapter.

Appendix A.

In order to apply the moment restictions (4.2.2)-(4.2.6), the set of instruments used in the estimation procedure must be specified. The following variables were included as instrument (note that this implies $z^1_{h,t} = \dots = z^5_{h,t}$):

- constant term;
- one period lagged expenditure on food and holiday respectively;
- one period lagged price of food for the basic model;
- degree of urbanisation;
- region;
- province;
- social class;
- number of household members older than 11;
- number of children between 0 and 6;
- number of children between 7 and 11;
- number of children between 12 and 17;
- number of children older than 18.

Because the demographic variables are reported only once a year, and since the changes of these variables over time is limited, they were kept constant over the complete survey period. That is, the instruments were given the value reported by the household in the first month it participated in the panel.

The following values are possible for the variables degree of urbanization, region, province and social class:

- degree of urbanisation:
  1 = villages with more than 50 % agrarians;
  2 = villages whith between 40 and 50 % agrarians;
  3 = villages with between 30 and 40 % agrarians;
  4 = villages with between 20 and 30 % agrarians;
  5 = industrialized rural villages with less than 5,000 inhabitants;
  6 = industrialized rural villages with between 5,000 and 20,000 inhabitants;

7 = commuter suburbs;
8 = small cities, with between 2,000 and 10,000 inhabitants;
9 = small cities, with between 10,000 and 30,000 inhabitants;
10= medium cities, with between 30,000 and 50,000 inhabitants;
11= medium cities, with between 50,000 and 100,000 inhabitants;
12= large cities, with more than 100,000 inhabitants;
13= Amsterdam, Rotterdam, The Hague;

- region:
  1 = the 4 major cities (Amsterdam, Rotterdam, The Hague and Utrecht);
  2 = remainder of western part of the Netherlands (except 1 and 6);
  3 = northern part of the Netherlands;
  4 = eastern part of the Netherlands;
  5 = southern part of the Netherlands;
  6 = suburbs of the 4 major cities;

- province:
  1 = Groningen;
  2 = Friesland;
  3 = Drenthe;
  4 = Overijssel;
  5 = Gelderland;
  6 = Utrecht;
  7 = Noord Holland (except 12);
  8 = Zuid Holland (except 12);
  9 = Zeeland;
  10= Noord Brabant;
  11= Limburg;
  12= Amsterdam, Rotterdam, The Hague;
  13= Flevoland;

- social class:
  5 = upper class;
  4 = upper middle class;
  3 = middle class;
  2 = lower middle class;

1 = lower class.

Because the differences between the different values of the urbanization variable are minor, the models were also estimated using a less detailed urbanization variable as instrument. The value one of this new variable corresponds to the values one to five of the old one, the value two to the the values six to ten, the value three to the values eleven and twelve and the value four to the value thirteen. Moreover, because the variables region and province are correlated (though not perfectly), the models of section 4 were also reestimated excluding the province variable from the instrument set. Both these changes did not alter the outcome of the estimation process in any significant way.

APPENDIX B.

General formulation of the moment restrictions used for the food case (the numbers with which they are indicated correspond with those in section 4):

intratemporal:

$$E\,\{[\frac{\partial u(q_{h,t})}{\partial q_{h,t,1}}\cdot\frac{1}{p_{t,1}} - \frac{\partial u(q_{h,t})}{\partial q_{h,t,2}}\cdot\frac{1}{p_{t,2}}]\cdot z^{1}_{h,t}\} = 0 \qquad (4.2.4')$$

for $t=1,\dots,41$

intertemporal:

$$E\,\{[\frac{\partial u(q_{h,t})}{\partial q_{h,t,1}}\cdot\frac{1}{p_{t,1}} - (\frac{1+r}{1+\rho})\cdot\frac{\partial u(q_{h,t+1})}{\partial q_{h,t+1,1}}\cdot\frac{1}{p_{t+1,1}}]\cdot z^{2}_{h,t}\} = 0 \qquad (4.2.5')$$

for $t=1,\dots,41$

$$E\,\{[\frac{\partial u(q_{h,41})}{\partial q_{h,41,1}}\cdot\frac{1}{p_{t,41}} - (\frac{1+r}{1+\rho})\cdot\frac{\partial u(q_{h,42})}{\partial q_{h,42,2}}\cdot\frac{1}{p_{42,2}}]\cdot z^{3}_{h,41}\} = 0 \qquad (4.2.6')$$

As already noted in section 3, this is just one of the systems of moments that can be derived from the first order conditions. For instance, it is possible to replace (4.2.6') by the intratemporal moment corresponding to period 42, or by the intertemporal moment for the second good corresponding to the periods 41 and 42. If the model is correctly specified, the estimation results should not be affected too much by such changes.

The general formulation of the moment restictions used for the holiday case can be written as follows:

$$E\,\{[\frac{\partial u(q_{h,t})}{\partial q_{h,t,1}}\cdot\frac{1}{p_{t,1}} - (\frac{1+r}{1+\rho})\cdot\frac{\partial u(q_{h,t+1})}{\partial q_{h,t+1,1}}\cdot\frac{1}{p_{t+1,1}}]\cdot z^{4}_{h,t}\} = 0 \qquad (4.2.7')$$

$$E\,\{[\frac{\partial u(q_{h,t})}{\partial q_{h,t,1}} - \frac{\partial u(q_{h,t})}{\partial q_{h,t,2}}\cdot I_{(0,\infty)}(q_{h,t,2}) - (\frac{1+r}{1+\rho})\cdot \frac{\partial u(q_{h,t+1})}{\partial q_{h,t+1,1}}\cdot(\frac{p_{t,1} - p_{t,2}\cdot I_{(0,\infty)}(q_{h,t,2})}{p_{t+1,1}})]\cdot z^{5}_{h,t}\} = 0 \qquad (4.2.8')$$

for $t=1,\dots,41$

Notes to chapter 2.

1 For example, in many cases exactly the same deterministic identities must be satisfied by all observations in the data set. However, because these identities (often) are functions of the consumed quantities which differ across observations, it is very unlikely, or even impossible, that all of these identities are satisfied for any particular choice of parameter values.

2 Only the relevance of these two approaches with respect to avoiding intratemporal deterministic relations is considered. Other reasons for using either one of these approaches are neglected.

3 For example, replacing the utility function used by both MaCurdy (1983) and Altonji and Siow (1987) by another specification, such as L.E.S. or a quadratic one, and repeating their analysis, may prove to be difficult.

4 This may be a reason why in many studies such restrictions are not included. For instance, MaCurdy (1983) limits his attention to the employed. However, extending his analysis to the unemployed (which does not seem to be a far-fetched generalization) may not be a straightforward excercise.

5 Notice that as a consequence also total consumption will not be known at the beginning of a period. This implies that two stage budgeting is no longer possible.

6 This ordering need not be the same for all consumers.

7 A similar argument, in a somewhat different context, is also given by Deaton (1977).

8 This exogeneity assumption, which might be considered to be strong, is usually imposed (explicitly or implicitly) in studies of the life cycle model under uncertainty.

9 The interest rate r_τ is assumed to be uncertain during period τ. It is assumed to be realized at the beginning of period $\tau+1$. Furthermore, if $u_\tau(q_\tau)$ is equal to, say, $u(q_\tau, z_\tau)$, with z_τ a vector of taste shifters, the set of period τ may be transformed into

$$\{y_{t+1}, p_{t+1}, z_{t+1}, r_t, \ldots, y_\tau, p_\tau, z_\tau, r_{\tau-1}\}.$$

10 In order to be able to apply Neustadt (1976, ch. III), one has to choose some underlying vector space. Melenberg and Alessie (1989) have chosen $q_{m\tau}$ to be an element of $L(V_{m\tau}, \mathbb{R})$, the set of functions with domain $V_{m\tau}$, consisting of possible outcomes of

$$(\bar{\eta}'_t, \eta'_{t+1}, \ldots, \eta'_{\tau-1}, \underline{\eta}'_\tau, \eta'_{m\tau})',$$

and range $\mathbb{R}$. In order to avoid measure theoretical problems they restricted $V_{m\tau}$ to be finite. Once $L(V_{m\tau}, \mathbb{R})$ has been chosen as the linear space that includes $q_{m\tau}$, the application of Neustadt becomes more or less straightforward. See Melenberg and Alessie (1989) for details.

11 The model can easily be extended to deal with more than two goods.

12 Quite similarly one can obtain a system of Euler equations relating two arbitrary periods τ and $\tau+1$ on the basis of period t's model formulation. Choose $h_{\tau+1}$ such that $h_{\tau+1} = Diag_{\tau,\tau+1} \cdot (-h_\tau)$, where

$$Diag_{\tau,\tau+1} = diag([i_{t\tau} p_{1\tau} / i_{t,\tau+1} p_{1,\tau+1}], \ldots, [i_{t\tau} p_{M\tau} / i_{t,\tau+1} p_{M,\tau+1}]),$$

and choose the other h's equal to zero. Then one obtains

$$E_t\{[Diag_{\tau,\tau+1} Du_{\tau+1}(q_{\tau+1}) - Du_\tau(q_\tau)]' h_\tau\} =$$

$$E_t\{E_\tau\{[Diag_{\tau,\tau+1} Du_{\tau+1}(q_{\tau+1}) - Du_\tau(q_\tau)]' h_\tau\}\} = 0,$$

where E_τ denotes the conditional expectation, conditional upon what is known at the beginning of planning period τ. Then by choosing

$$h_\tau = E_\tau[\mathrm{Diag}_{\tau,\tau+1} Du_{\tau+1}(q_{\tau+1}) - Du_\tau(q_\tau)]$$

we get

$$E_\tau[\mathrm{Diag}_{\tau,\tau+1} Du_{\tau+1}(q_{\tau+1}) - Du_\tau(q_\tau)] = 0.$$

13 This choice for h_{2t} is allowed since q_{2t} is a function of the right input variables, i.e., q_{2t} only depends upon the input variable h_{2t} is allowed to depend upon.

14 Some households enter the panel in the first month but leave before September 1987, whereas other households enter the panel in later months.

15 Generally, the first order conditions can also be combined into restrictions linking non-consecutive periods. Such restrictions are neglected in this study.

16 This particular normalization is chosen because it implies that all that remains to be checked to ensure the concavity of the utility function, is whether the parameter c is negative.

17 The bliss points are $b \cdot e - c \cdot d$ for the first good, and $b \cdot d - e \cdot (1+b^2)/c$ for the second one.

18 The computational difficulties arose when trying to determine the inverse of the outer product of the vector of moment restrictions, which is necessary in order to determine the optimal weighting matrix. Although this matrix should be positive semidefinite, it turned out not to be so. Subsequent computation of the eigenvalues of this matrix, indicated that some of them were very close to zero, but negative. Given the size of the negative eigenvalues, it was concluded that this problem was due to rounding errors.

19 There is also a practical reason for doing this, since if the moment restictions are not averaged over time, there would be, depending on

which case is considerd, 996 or 984 of these restrictions. Obtaining efficient GMM estimates requires a square matrix weighting the moments. In order to determine this matrix of dimension 996×996 or 984×984, a matrix of the same dimension must be inverted (cf., Hansen and Singleton(1982)). However, the mainframe on which the computations for this study were performed (a VAX 8700), did not allow for matrices of such a dimension.

20 In the case of the inclusion of the one period lagged price of holidays in the instrument set, the iterative procedure used to determine consistent estimates, which are needed for constructing the optimal weighting matrix, did not converge within acceptable time limits.

21 Although (quasi-)concavity of the utility function is usually required in models of consumer behaviour, it is not always found in empircal work. See, for example, Hansen and Singleton (1984).

22 Observations not satisfying this requirement are incompatible with the assumed rational behaviour of consumers, as the same expected utility level can be obtained from a lower consumption level.

23 For the sake of completeness, it was also checked whether the intratemporal equations held exactly, as they should if the standard life cycle model were to be correct. Not surprisingly, this was not the case.

CHAPTER 3

LARGE, INFREQUENT CONSUMPTION IN THE MULTI-GOOD LIFE CYCLE CONSUMPTION MODEL

1. Introduction.

In empirical studies applying a life cycle framework for modelling the behaviour of consumers, different types of data sets are employed. As was already noted in chapter 1, some studies are concerned with the life cycle hypothesis at the macro level. Hence, in these studies macroeconomic quantities, usually in per capita terms, are used. In order to justify the use of macroeconomic data for estimating what are essentially microeconomic models, these studies usually have to impose the well-known 'representative consumer' assumption. Examples of this approach can be found in Hall (1978, 1988), Hansen and Singleton (1982, 1983), Flavin (1981) and Bean (1986). Since the focus in this study is on the life cycle model at the micro level, data on a corresponding level are needed. Put more precisely, as the multi-good version of the life cycle model is considered, household expenditures disaggregated into several commodity categories are required. Examples of studies using such data are the contributions of Alessie, Kapteyn and Melenberg (1988), Alessie and Kapteyn (1989) and Blundell, Browning and Meghir (1988).

In this chapter, a problem that may occur if one uses such a disaggregated data set for estimating a life cycle model is studied. If the data set is sufficiently disaggregated over goods as well as over time[1], it often will contain one or more goods which for most households display a strongly fluctuating expenditure pattern. Typically, such a commodity is either not *bought* at all in a particular period or, if it is *bought*, it is in (relatively) large quantities only. One possible explanation for this pattern can be found in the so-called 'infrequency of purchase' literature[2] (see, for instance, Deaton and Irish (1984) or Pudney (1989) chapter 4), which explains the existence of such expenditure patterns from the observation that consumption and expenditures may differ substantially, if the period over which they are measured is short. The line of reasoning followed in studies belonging to this strand of literature, can briefly be illustrated by the following example. Consider a household consuming a certain commodity every week, but buying it only once every fortnight. If the observation period is a week, the household will either be observed not to buy the good at all, or to buy twice the quantity that in reality is consumed during the observation period.

Although the distinction between consumption and expenditures that is made in these studies, in many cases can provide a satisfactory explanation for the aforementioned fluctuating expenditure patterns, in some situations it may be worthwhile to consider an alternative explanation. This is, for example, the case if the fluctuating expenditure pattern for a particular good corresponds with fluctuations in the underlying consumption behaviour. That is, such a good is either not *consumed* at all, or is *consumed* in relatively large quantities only. A typical example of such a good is the vacation of households.

To illustrate this, consider the data set used by Van Soest and Kooreman (1987) in their study on vacation behaviour. The distribution of the (positive) annual expenditures on vacations as reported in this data set, is given in Table 1.[3] From this table it can be seen that, for example, less than five per cent of the reported vacation expenditures are below Dfl 300.- (currently about $ 150.-).

An example in a somewhat different context can be found in some studies on labour supply. In these studies, it often is observed that people either do not work at all, or work a considerable number of hours. Hausman (1980) and Cogan (1981), for example, study this phenomenon using a static framework.

Table 1: Distribution of positive holiday expenditures[1)]

AMOUNT	PERCENT
0-100	1.2
100-200	2.0
200-300	1.4
300-400	3.0
400-500	3.1
500-600	4.1
600-700	3.8
700-800	4.3
800-900	4.3
900-1000	4.7
>1000	68.1
#obs	1143

1) source: 1815 household observations from the 1981 Consumer Expenditure Survey of the Netherlands Bureau of Statistics
AMOUNT = average annual expenditure on vacation (in Dutch guilders)
PERCENT = number of positive expenditures on vacations in a certain class as a percentage of the total number of positive expenditures on vacations
#obs = total number of positive vacation expenditures

The aim of this chapter is to explain such fluctuating patterns in a dynamic context: the life cycle model. In order to illustrate the model which is developed to offer this explanation,the vacation example is used. Applying the model to the labour supply case does not seem to be more difficult, but is not done here because of lacking labour supply data. The content of this chapter can briefly be summarized as follows.

In section 2, it is shown that a strongly fluctuating consumption pattern cannot be fully explained within the multi-good version of Hall's (1978) life cycle model discussed in the previous chapter. Therefore, a modification of this model is proposed, and its consequences are examined. The essential feature of this modified model is that either the preference ordering, or the budget constraint is non-convex for small values of the good displaying the strong consumption fluctuations. In section 3, this modified life cycle model is estimated and tested, using the panel which was already employed in the previous chapter. The estimation procedure which is used to do this, is also taken from the previous chapter. Finally, some concluding remarks are made in section 4.

2. Modelling Infrequency of Consumption.

Consider the following two-good version of Hall's (1978) life cycle consumption model under uncertainty, which was already discussed in the previous chapter (for t=1,...,L):

$$\max_{x_t, y_t, \dots x_L, y_L} \quad E_t \sum_{\tau=t}^{L} \left(\frac{1}{1+\rho}\right)^{\tau-t} \cdot u(x_\tau, y_\tau) \tag{2.1}$$

$$\text{s.t.} \sum_{\tau=t}^{L} \left(\frac{1}{1+r}\right)^{\tau-t} \cdot [p_{\tau x} x_\tau + p_{\tau y} y_\tau] \le W_t = (1+r)A_{t-1} + \sum_{\tau=t}^{L} \left(\frac{1}{1+r}\right)^{\tau-t} \cdot i_\tau,$$

$$x_\tau,\ y_\tau \ge 0 \quad \tau = t, \dots, L^{4},$$

where

$u(\cdot)$	: within period utility function; assumed to be strictly concave, constant over time and increasing in its arguments,
$(x_\tau, y_\tau)'$	: period τ's consumption vector,
$(p_{\tau x}, p_{\tau y})'$	: period τ's price vector,
i_τ	: period τ's income,
r	: nominal interest rate; assumed to be constant over time,
ρ	: time preference parameter,
A_{t-1}	: assets available at the beginning of period t,
E_t	: expectation conditional on the information available at period t.

Because of the multi-good setting of this model, prices must, in contrast with Hall's model, be included. This implies that uncertainty in model (2.1) may result not only from future incomes, as in Hall's model, but also from future prices. Consider now the example on which will be the focus in the empirical part of this study: the monthly consumption of vacation. A household typically will not go on vacation every month, but will only take one or two vacations per year, during which relatively large amounts of money will be spent. Can the model given in (2.1) explain such consumer behaviour, implying either a considerable consumption level, or no consumption at all?

As the interest rate, the time preference parameter and the preference ordering over all possible commodity bundles within a period, all are assumed to be constant over time, they cannot account for the variation over time of the consumption level. Since the life cycle model was especially formulated to account for the effect that an income change in a period is smoothed over several periods, the only possible cause left for explaining the jump from a substantial consumption level in one month, to no consumption in the next (and vice versa), is a big shift in the price of vacation. However, as can be seen from Table 2, the monthly price variability during the period covered by the data set used in this study is very limited, both in absolute and in relative terms. So, unless the own price elasticity is very large, prices cannot fully account for the large changes in the consumption level of vacation.

Moreover, in months during which many households report holiday expenditures (i.e. the holiday season from May until September)[5], the price of holidays often rises more than the price of the other commodity (compare %ΔPV with %ΔPNV in Table 2). This combined increase in the price of holidays (albeit small) and the number of households spending money on vacations, cannot be explained by the model given in (2.1), unless the own price elasticity of vacation is positive.

Given the limitations of model (2.1), these findings are not very surprising. There are several ways in which model (2.1) can be changed, so as to better explain the fluctuating consumption pattern. A straightforward generalization is to make the within period utility function $u(\cdot)$ time specific (for example by including taste shifters), thus capturing the seasonal pattern in the number of households reporting holiday expenditures present in Table 2. However, although this approach is likely to generate a more fluctuating consumption pattern, it is not fully satisfactory, as it still does not exclude the consumption of small but positive quantities in general.

Table 2: Price variability and purchase frequency of vacation[1)]

Period	NH	%NZ	RPV	%ΔPV	%ΔPNV	Period	NH	%NZ	RPV	%ΔPV	%ΔPNV
Apr 84	921	21.2	1.00	.	.	Jan 86	676	15.8	1.02	-0.1	-0.5
May 84	966	25.9	1.00	0.0	0.1	Feb 86	667	16.8	1.01	0.0	0.1
Jun 84	884	33.4	1.00	0.5	0.0	Mar 86	680	17.3	1.01	0.0	0.2
Jul 84	922	40.8	1.01	0.1	-0.1	Apr 86	706	21.9	1.01	0.6	0.3
Aug 84	855	31.7	1.00	0.0	0.1	May 86	676	29.0	1.01	-0.3	0.0
Sep 84	757	19.5	1.00	-0.2	0.5	Jun 86	776	30.1	1.02	0.3	-0.5
Oct 84	889	14.1	1.00	0.8	0.6	Jul 86	818	40.2	1.03	-0.1	-1.0
Nov 84	849	9.5	1.00	0.1	0.1	Aug 86	798	30.7	1.03	0.1	0.1
Dec 84	789	10.8	1.00	0.0	-0.1	Sep 86	787	19.2	1.02	0.1	0.5
Jan 85	736	14.3	1.00	0.2	-0.1	Oct 86	837	16.8	1.02	-0.1	0.7
Feb 85	693	17.9	1.00	0.0	0.5	Nov 86	858	9.8	1.01	-0.4	-0.1
Mar 85	856	17.1	0.99	-0.3	0.5	Dec 86	978	10.7	1.02	0.8	-0.1
Apr 85	816	22.4	1.00	1.7	0.4	Jan 87	956	15.9	1.04	-0.1	-1.6
May 85	751	28.3	1.01	0.7	0.1	Feb 87	1022	16.5	1.03	0.0	0.3
Jun 85	753	28.8	1.01	0.1	-0.1	Mar 87	1018	19.4	1.03	-0.2	0.0
Jul 85	757	37.0	1.02	0.2	-0.2	Apr 87	981	21.5	1.04	1.2	0.5
Aug 85	767	29.0	1.01	0.0	0.1	May 87	1024	28.5	1.04	-0.1	-0.1
Sep 85	789	20.0	1.01	-0.2	0.4	Jun 87	1052	33.5	1.03	-0.2	0.1
Oct 85	806	14.0	1.01	0.6	0.3	Jul 87	968	39.4	1.04	0.1	-0.1
Nov 85	764	8.6	1.01	-0.1	0.0	Aug 87	954	33.6	1.04	0.7	0.2
Dec 85	742	9.8	1.01	-0.1	-0.2	Sep 87	898	23.2	1.04	0.0	0.4

1) NH = number of households participating in the panel in a particular month
%NZ = percentage of these households reporting positive expenditures for vacation in that month
%ΔPV = monthly percentage change in the price index of vacation; the price of vacation in April '84 has been set equal to 100
%ΔPNV = monthly percentage change in the price index of nonvacation good; the price of non-vaction in April '84 has been set equal to 100
RPV = price of vacation relative to the price of the nonvacation good

A second way in which model (2.1) can be improved, is by no longer maintaining the assumption that the lifetime utility function is additively separable over time. With a commodity like vacation, it is possible that consumption in a particular month will influence utility in a number of preceding and subsequent months. A possible way of modelling this, is to assume that by going on vacation, one builds up a stock of 'holiday pleasure'. This stock renders utility not only during the holiday itself, but also in a number of preceding and subsequent months. As time goes by the stock decreases (for example because of the daily routine at work), until a certain minimum level is reached, at which point the household replenishes the stock by going on holiday again. A problem with this approach in empirical work, is that one has to construct the (usually unobserved) stock of 'holiday pleasure'. Moreover, this modification again does not exclude the possibility that households, when replenishing their stock, do so by consuming only a small quantity of vacation.

As both modifications of model (2.1) discussed so far do not exclude low consumption levels for vacation, a third alternative is considered. In this approach, either each period's preference ordering or cost structure is changed in such a way, that consuming small quantities in any period does not result in the maximum expected utility.[6]

There are several possible motivations for a preference ordering which would imply that the consumption of a small quantity of vacation in a certain period gives less expected utility than not going on holiday, and spending the money thus saved on other goods in that period, or use it for consumption in other periods. One such motivation could be that a holiday must span a certain minimum period, in order to enjoy it. Therefore, one prefers, for example, a fortnight's holiday to fourteen holidays of one day.

This preference ordering will be modelled below by introducing a transformation in the utility function which results in non-convex preferences for small values of the vacation commodity. But before turning to this, consider first the two simple examples of such a preference ordering depicted in Figures 1 and 2. In Figure 1, it is assumed that the consumption levels in all periods except period t remain unchanged. The preference ordering in this figure implies that by going from a low consumption level of the vacation commodity y (point A), along the budgetline to no consumption of this good (point B), a higher utility level can be reached.

Instead of using the money that becomes available by not going on vacation in period t on the other good in the same period, it may be more plausible to assume that this money is used for consumption of good y in other periods. To illustrate this case, consider the example given in Figure 2. Assume that the consumer has perfect foresight and only varies the consumption of good y in period t and t+1. And again, as Figure 2 shows, consuming small quantities of good y in period t or t+1 (points A and C), results in a lower expected utility level, than consuming not going on vacation in either of these periods, and spending the money thus saved on good y in the other period (points B and B').

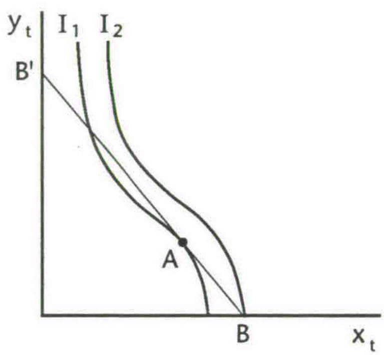

Figure 1

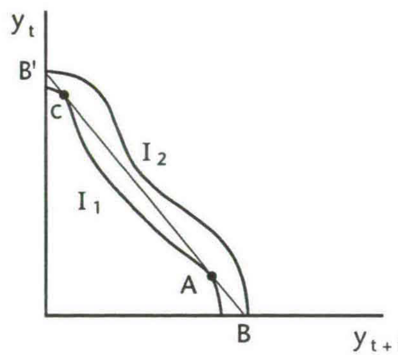

Figure 2

I_i = i - th indifference curve; i=1,2

BB' = budgetline

In both these examples only the consequences of shifts of money from one good to one other good, keeping all other consumption levels unchanged, were considered. Of course, much more complicated transfers are possible, but they cannot easily be represented in simple diagrams. More importantly, the main point of the two examples is not to demonstrate all possible ways in which the money that becomes available by not consuming good y can be redistributed, but to show that the proposed change of the preference ordering implies that a higher utility level can be reached by shifting consumption of good y towards zero in some periods.

In both examples, the crucial characteristic of the within period preference ordering is that it is no longer convex for small values of y. As convexity of the preference ordering is equivalent to quasi-concavity of the utility function (see e.g. Deaton and Muellbauer (1980) page 30), this change in the preference ordering can be incorporated in model (2.1) by changing the strict concave period utility function $u(\cdot)$ in such a way, that it is not quasi-concave for small values of y. This can be achieved by replacing y_τ in $u(\cdot)$ by a transformed value $g(y_\tau)$, with $g(\cdot)$ a strictly increasing function which is strictly convex for small values of y_τ, and concave for larger values. An example of such a transformation is the well-known logistic function.

Because of the strict concavity of $u(\cdot)$ with respect to x_τ and y_τ[7], this convexity of $g(\cdot)$ itself does not imply that $u(\cdot)$ is no longer quasi-concave for small values of y_τ. Using the necessary second order conditions for quasi-concavity of $u(\cdot)$ (see for example Takayama (1974) page 123), a sufficient condition on the transformation $g(\cdot)$ guaranteeing non-convex preferences for small values of y_τ can be derived. It states that, given a value of x_τ, the following must hold for values of y_τ smaller than $\bar{y}_\tau$ (defined below):[8]

$$g''(y) > h(x,y) \equiv \{[-u_{xx}u_g^2 - u_{gg}u_x^2 + 2\,u_x u_g u_{xg}]\cdot (g'(y))^2 / [u_g u_x^2]\} > 0 \qquad \text{for } y \leq \bar{y}, \tag{2.2}$$

where

u_i = partial derivative of $u(x,g)$ with respect to i; i=x,g,

u_{ij} = second order partial derivative of $u(x,g)$ with respect to i and j; i,j=x,g,

$g'(y)$ = first order derivative of $g(y)$,

$g''(y)$ = second order derivative of $g(y)$,

$\bar{y}$ = largest value of y satisfying condition (2.2).

Because $u(\cdot)$ is assumed to be strictly concave in x and g[9], and increasing in its arguments, the right hand side of (2.2) must be greater than zero. Hence condition (2.2) simply states that the convexity of the transformation $g(\cdot)$ must outweigh the concavity of the utility function $u(\cdot)$, in order to ensure that the modified model can account for the consumption pattern of goods like vacation. To recapitulate, the transformation $g(\cdot)$ is assumed to have the following properties, given a value of x:

$$\begin{aligned} &g'(y) > 0 \\ &g''(y) > h(x,y) \text{ if } y \leq \bar{y} \\ &g''(y) \leq h(x,y) \text{ if } y > \bar{y} \end{aligned} \tag{2.3}$$

An alternative way of introducing the modification in model (2.1) does not deal with the utility derived from a vacation, but with the costs associated with it. In model (2.1) it is assumed that the costs of a holiday increase proportionally to the quantity bought. However, for most holidays substantial costs must be incurred, irrespective of the quantity consumed. For example, whether one is one or two weeks on holiday has few consequences for the (often substantial) travelling expenses one has to make in order to get to one's holiday residence.

The presence of such 'initial costs', imply that if one increases the quantity consumed, the average costs will diminish, but at a decreasing rate. Eventually, this process may be stopped or even reversed as a number of restrictions (time available for holidays, duration of reservations, or package tours) become binding, implying constant or even increasing average costs from this point onwards.

This change in the cost structure can be incorporated in model (2.1), by replacing y_τ in the budget constraint by the transformed quantity $f(y_\tau)$, where $f(\cdot)$ is assumed to be strictly increasing, strictly concave for small values of y, and convex for larger values. This model can be considered as a continuous and differentiable version of the well-known fixed costs model. Static versions of this model have been used in labour supply studies see, for example, Hausman (1980) and Cogan (1981).[10]

An example of the transformation introduced above, is the inverse of the logistic function. The consequences of including such a transformation in the lifetime budget constraint, are illustrated in Figure 3. In this illustration it is assumed that the consumption of all periods except period t remains unchanged. As can be seen from this figure, setting the consumption of y in period t equal to zero, i.e. going from point A to point B, increases the expected lifetime utility.

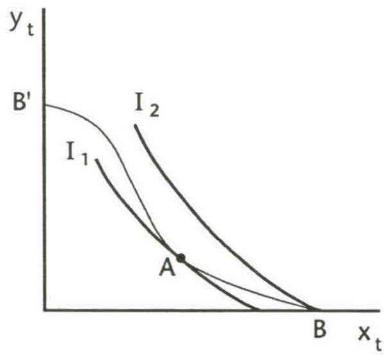

Figure 3

I_i = i - th indifference curve; i=1,2

BB' = budgetline

It can easily be demonstrated that, although the two proposed modifications (changing the preference ordering, and the cost structure respectively) result from two different lines of reasoning, the resulting models are equivalent in the sense that given a function $f(\cdot)$, one can always find a corresponding function $g(\cdot)$. In order to demonstrate this, consider the life cycle model in which the first modification is incorporated:

$$\max_{x_t,y_t,..x_L,y_L} E_t \sum_{\tau=t}^{L} \left(\frac{1}{1+\rho}\right)^{\tau-t} \cdot u(x_\tau, g(y_\tau))$$

$$\text{s.t. } \sum_{\tau=t}^{L} \left(\frac{1}{1+r}\right)^{\tau-t} \cdot [p_{\tau x} x_\tau + p_{\tau y} y_\tau] \le W_t, \tag{2.4}$$

$$x_\tau,\ y_\tau \ge 0 \quad \tau = t,\ldots.,L.$$

Next define y^*_τ to be equal to $g(y_\tau)$, and substitute this in model (2.4):

$$\max_{x_t, y_t^*, \ldots x_L, y_L^*} E_t \; \sum_{\tau=t}^{L} \left(\frac{1}{1+\rho}\right)^{\tau-t} \cdot u(x_\tau, y_\tau^*)$$

$$\text{s.t.} \; \sum_{\tau=t}^{L} \left(\frac{1}{1+r}\right)^{\tau-t} \cdot [p_{\tau x} x_\tau + p_{\tau y} g^{-1}(y_\tau^*)] \le W_t, \tag{2.5}$$

$$x_\tau, \; g^{-1}(y_\tau^*) \ge 0 \quad \tau = t, \ldots, L.$$

Because of the assumed shape of the function $g(\cdot)$, its inverse, say $f(\cdot)$, is a function that is concave for small values of y_τ^*, and convex for larger values. So, as was claimed, model (2.5) is just the life cycle model incorporating the second modification.

Because of this equivalence, the strict concavity of $u(\cdot)$ again makes the imposition of an additional condition on $f(\cdot)$ necessary, to ensure that small quantities of y will not be chosen. This condition can be derived either from model (2.5) directly, or, because of the aforementioned equivalence, from the condition on $g(\cdot)$ given in (2.2). Following this second approach, it is straightforward to show that condition (2.2), given the properties (2.6)-(2.8), is equivalent to condition (2.9) given below.

$$\frac{\partial u(x_\tau, y_\tau^*)}{\partial y_\tau^*} = \frac{\partial u(x_\tau, g(y_\tau))}{\partial g} \tag{2.6}$$

$$g'(y_\tau) = [f'(y_\tau^*)]^{-1} \tag{2.7}$$

$$g''(y_\tau) = -[g'(y_\tau)]^2 \cdot f''(y_\tau^*)/f'(y_\tau^*) \tag{2.8}$$

$$f''(y_\tau^*) < [u_{xx} u_{y*}^2 + u_{y*y*} u_x^2 - 2u_x u_{y*} u_{xy*}] \cdot$$
$$f'(y_\tau^*)/\,[u_x^2 u_{y*}] < 0 \qquad \text{if } y_\tau^* \le g(\bar{y}_\tau) \tag{2.9}$$

Condition (2.9) simply states that, in order to guarantee that no small quantities of good y_τ^* are chosen, the concavity of $f(\cdot)$ must outweigh the concavity of $u(\cdot)$ for these values of y_τ^*. As each of the models (2.4) and (2.5) can be written in terms of the other one, it suffices to study either one of them. In the remainder of this chapter the modified model given in (2.4) is considered.

The usual conditions which guarantee the existence (and uniqueness) of a solution which, moreover, is fully characterized by the first order conditions, are not satisfied for model (2.4), since by incorporating the transformation $g(\cdot)$ the lifetime utility function is no longer strictly concave. In appendix A of this chapter, conditions ensuring the existence of a solution which is characterized by the first order conditions are given. The only problem remaining is that the solution need not be the only commodity bundle satisfying the first order conditions, as is illustrated, for example, in Figure 1. The assumptions made in this example imply that point B results in the highest expected lifetime utility. Hence, if a consumer behaves rationally, which is an assumption underlying the life cycle model, he or she will choose point B. Thus, only point B is observed by the researcher.

There is, however, one situation in which the possibility of multiple solutions might cause a problem, namely if there is a future period in which two different commodity bundles, adding up to the same period consumption, result in the same maximum (expected) period utility. In this case, one might be confronted with a so-called time consistency problem, as a consumer can *plan* in period t to consume one commodity bundle in this future period, but can actually *realize* the other bundle without changing the expected lifetime utility. As a result, the modified life cycle model (2.4) is still valid in planned quantities, but may no longer be valid in the corresponding realizations (see Melenberg and Alessie (1989) for a more general discussion on time consistency problems).

Figure 4 shows for a certain realization of the variables influencing period τ's ($\tau > t$) consumption, i.e., period τ's input variables, that such a situation can occur in model (2.4), since points B and C result in the same maximum utility in this period. As most data sets do not contain information on the consumption plans of households, this could seriously restrict the empirical usefulness of model (2.4). However, it is not very likely that a situation as depicted in Figure 4 will often occur. For example, any change in the price of y_τ relative to the price of x_τ changes the slope of the budgetline, resulting in different utility levels for the interior and the corner solution. So, for exactly one ratio of period τ's prices, given the values of the other variables influencing period τ's consumption, this time consistency problem can occur.

In order to exclude this unlikely event, not only the usual time consistency conditions (cf. Melenberg and Alessie (1989)) must hold, but an additional condition is needed. The additional time consistency condition imposed here, is that if the above described situation occurs, a household does not deviate from its original consumption plan when arriving in period τ. Given that deviating from the plan does not yield extra utility for the consumer, and the fact that the time consistency problem occurs only for particular values of the input variables, this additional assumption seems not to be to restrictive.

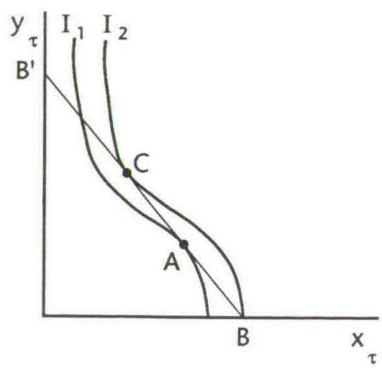

Figure 4

I_i = i - th indifference curve; i=1,2

BB' = budgetline

3. Empirical Application.

3.1 Specification and data.

In the empirical application considered in this section, the model given in (2.4) will be estimated using as commodities vacation and non-vacation. The further specification of this model, necessary to enable estimation, is taken from the previous chapter. That is, $u(\cdot)$ is assumed to be quadratic with respect to $x_{h,\tau}$ and $g(\cdot)$, and is made household specific by reparameterizing the parameters corresponding with the linear part of the utility function. The transformation $g(\cdot)$ is specified in such a way that the standard model, i.e., the model considered in the previous chapter in which the preferences are globally convex, is a special case. Hence, the specification used in this chapter can be summarized as follows:

$$u(x_{h,\tau},g(y_{h,\tau})) = \tfrac{1}{2}\{((1+b)^2/c)\cdot x_{h,\tau}^2+2\cdot b\cdot x_{h,\tau}\cdot g(y_{h,\tau})+ c\cdot g(y_{h,\tau})^2\} + d\cdot x_{h,\tau}+e\cdot g(y_{h,\tau}), \tag{3.1.1}$$

$$d_h = d_0+d_1\cdot \log(fs_h), \tag{3.1.2}$$

$$e_h = e_0+e_1\cdot \log(fs_h), \tag{3.1.3}$$

$$g(y_{h,\tau}) = y_{h,\tau}/(1+\beta\cdot \exp(-y_{h,\tau})). \tag{3.1.4}$$

where b, c, d_0, d_1, e_0, e_1 and β are parameters to be estimated, and fs_h is the household size of household h.

The estimation procedure employed in this chapter also stems from the previous chapter. That is, under the assumption of intratemporal uncertainty introduced in chapter 2, the first order conditions corresponding with model (2.4) can be combined in a system of inter- and intratemporal moment restrictions. Let $z_{h,\tau}$ denote the vector of instruments described in appendix A of the previous chapter. The system of moment restrictions for the model presently under consideration, can then be written as follows:

$$E\,[\{[(1+b^2)/c]\cdot[\frac{x_{h,t}}{p_{t,x}} - (\frac{1+r}{1+\rho})\cdot\frac{x_{h,t+1}}{p_{t+1,x}}] + b\cdot[\frac{g(y_{h,t})}{p_{t,x}} - (\frac{1+r}{1+\rho})\cdot\frac{g(y_{h,t+1})}{p_{t+1,x}}] + d\cdot[\frac{1}{p_{t,x}} - (\frac{1+r}{1+\rho})\cdot\frac{1}{p_{t+1,x}}]\}\ z_{h,t}] = 0, \tag{3.1.5}$$

$$E\ [\{[(1+b^2)/c]\cdot x_{h,t} + b\cdot g(y_{h,t}) + d -$$

$$g'(y_{h,t})\cdot(b\cdot x_{h,t} + c\cdot g(y_{h,t}) + e)\cdot I_{(0,\infty)}(y_{h,t}) -$$

$$(\frac{1+r}{1+\rho})\cdot\ ([(1+b^2)/c]\cdot x_{h,t+1} + b\cdot g(y_{h,t+1}) + d)\cdot$$

$$(\frac{p_{t,x} - p_{t,y}\cdot I_{(0,\infty)}(y_{h,t})}{p_{t+1,x}})]\}\ z_{h,t}] = 0, \qquad (3.1.6)$$

$$\text{for } t=1,\ldots,41.$$

The data used to construct the sample analogue of this system are the same as those employed in the previous chapter. Apart form the remarks concerning the data made in that chapter, there are some aspects which are especially relevant for the topic considered here. Firstly, as Tables 2 (on page 6) and 3 show, positive vacation expenditures are reported infrequently by households in all months. Secondly, Table 4 indicates that a relatively large proportion of the reported vacation expenditures concerns small amounts. This second finding would, at first glance, suggest that consumption levels of vacation can be low, thus contradicting earlier statements regarding the consumption pattern of vacation and, moreover, making the proposed transformation superfluous.

However, it is important to note that, due to the way in which they are collected, the data in the 'Intomart consumer expenditure panel' refer to the *expenditures* on vacation, whereas the model discussed thusfar is concerned with the *consumption* of vacation. Expenditures on and consumption of holidays are likely to differ substantially, if measured on a monthly basis. For example, one often has to pay a part of the expenses in advance (a ticket, a hotel reservation or a part of one's holiday equipment). Or a vacation can cover (parts of) two consecutive months, which might result in the reporting of vacation expenditures in both months. In this case, the data suggest that there were two separate holidays.

Moreover, the definition of the vacation good which was used when constructing this data set introduces an additional difficulty, as it includes day trips and school outings. This complicates matters, since a consumer when deciding on taking a day trip or going on a school outing is likely to take different aspects into consideration, than when deciding on taking a vacation which spans a longer period. Hence, if one wants to describe adequately the decision process regarding these longer holidays, as is the case in this study, it should be clearly separated from other choices. The data used for estimating such a model should reflect this distinction. An example of a data set meeting this requirement is the one employed by Van Soest and Kooreman (1987). The definition of the vacation good used there requires that one stays away from home for recreational purposes for at least four succesive nights.

Unfortunately, the data set used in this latter study is a cross section, making it unsuited for estimating the complete dynamic model considered here. The way in which it could be used to estimate a part of the model, as well as the problems associated with it, are discussed briefly in appendix B. Because the 'Intomart consumer expenditure panel' introduced in the previous chapter does allow for the estimation of the full dynamic model, it will be used in the empirical application. In order to take account of the possibility that the difference between consumption and expenditures could influence the estimation results, three possible links between consumption and expenditures are considered.[11]

The first one corresponds to the assumption that is usually made, explicitly or implicitly, i.e., that the expenditures are an approximation of the corresponding consumption, close enough to allow model (2.4) to be formulated in expenditure terms.

The remarks made earlier, indicate that this assumption might not be appropriate in the case considered here. Therefore, a second link is considered which differs from the first one in that only outlays exceeding Dfl. 100.- are considered to represent vacation consumption. Expenditures below this amount are assumed to be the result of day trips or school outings. Since these activities are assumed not to come under the definition of the vacation good, the corresponding expenditures are removed from the data set by setting them equal to zero.[12]

In the third alternative it is assumed that the vacation expenditures made over a period of three months all correspond to one and the same vacation. This case is considered in order to take account of the aforementioned difference in timing of the consumption of and the payment for a vacation. This is done by replacing the monthly vacation expenditures by a three monthly sum. Put more precisely, if during three consecutive months a particular household reports positive holiday expenditures for at least two months, they are summed and attributed to the month in which the largest expenditures were reported. The holiday expenditures of the other month(s) are set equal to zero. From Table 5 it can be seen what effect this operation has on the data. Comparing this table with Table 4, it is clear that the share of small expenditures decreases, although it remains considerable, whereas the share of large expenditures increases. In section 3.2, the sensitivity of the estimation results with respect to the different assumptions is investigated. In the next chapter, the consequences of incorporating a link between consumption and expenditures in a life cycle model are studied in greater detail.

Table 3: Vacation expenditure frequency[1)]

NMONTHS

PERCENT	1-6	7-12	13-18	19-24	25-30	31-36	37-42
0-10	680	224	92	39	36	25	70
10-20	34	158	70	30	25	27	40
20-30	110	136	61	23	19	13	43
30-40	81	88	58	16	15	12	18
40-50	48	68	38	13	7	8	22
50-60	100	58	21	4	6	3	19
60-70	47	30	9	1	5	1	2
70-80	26	17	1	3	1	0	3
80-90	12	7	2	1	1	0	1
90-100	0	0	0	0	0	0	0
100	70	1	0	0	0	0	0

1) PERCENT = number of months a household spends money on vacation as a percentage of the total number of months a household participates in the panel.

NMONTHS = number of months a household participates in the panel.

Table 4: Distribution of positive vacation expenditures[1)]

AMOUNT	PERCENT	AMOUNT	PERCENT
0-50	18.0	550-600	2.6
50-100	10.9	600-650	2.0
100-150	7.6	650-700	1.9
150-200	6.5	700-750	1.4
200-250	5.9	750-800	1.7
250-300	4.9	800-850	1.3
300-350	3.3	850-900	1.1
350-400	3.4	800-950	1.0
400-450	3.0	950-1000	1.3
450-500	3.0	>1000	17.2
500-550	2.0	#obs	7762

1) AMOUNT = monthly expenditures on vacation (in Dutch guilders)
PERCENT = number of reported positive vacation expenditures in a certain class as a percentage of the total number of positive vacation expenditures
#obs = total number of positive vacation expenditures

Table 5: Positive three-monthly sum of vacation expenditures[1)]

AMOUNT	PERCENT	AMOUNT	PERCENT
0-50	10.9	550-600	2.4
50-100	8.2	600-650	2.0
100-150	6.2	650-700	2.1
150-200	5.0	700-750	1.6
200-250	4.9	750-800	1.9
250-300	4.2	800-850	1.5
300-350	3.2	850-900	1.7
350-400	3.6	800-950	1.7
400-450	3.0	950-1000	1.6
450-500	3.1	>1000	28.9
500-550	2.4	#obs	5050

1) AMOUNT = three-monthly sum of expenditures on vacation (in Dutch guilders)
PERCENT = number of reported positive vacation expenditures in a certain class as a percentage of the total number of positive vacation expenditures
#obs = total number of positive three monthly sums of vacation expenditures

3.2 Estimation results.

In Tables 6 and 7 the estimation results and test outcomes for the three data sets corresponding with different assumptions regarding the link between consumption and expenditures are presented for the basic and the household specific version, respectively.

The first aspect worth considering refers to the differences between the first two columns of each table. The first column of each table represents the results of the life cycle model without a transformation, i.e., the estimation results of the previous chapter. The second colummn of each table consists of the outcomes of the model with transformation (3.1.4), which are obtained using the original 'Intomart consumer expenditure panel'.

The comparison of the two columns of each table makes clear that the estimates of the parameters of the model of the previous chapter, i.e., all parameters except β, are not changed dramatically by the introduction of the transformation: the estimate of c increases somewhat (in absolute value), the estimate of $(1+r)/(1+\rho)$ remains practically the same, and the estimates of the other parameters become smaller. Furthermore, sign and significance of the estimates are (essentially) unchanged. The main consequence of the changes in the estimates is that the bliss point for the non-vacation good becomes substantially smaller. Hence, the number of non-vacation expenditures which are correctly located vis-à-vis the corresponding bliss point (cf. section 4.3 of the previous chapter for details on the bliss point condition) decreases considerably. This is especially true for the household specific version.

Apart from the changes in the value of the parameter estimates, the introduction of the transformation also influences the test outcomes. For the basic version, the test statistic of Hansen and Singleton's (1982) misspecification test (cf. chapter 2 for details) indicates that the model including the transformation is accepted, in contrast with the model of the previous chapter. The test results for the household specific version show that including the transformation improves the performance of the model, as one should expect, although the model without the transformation is accepted as well.

Turning next to the estimate of the parameter of the transformation, Tables 6 and 7 show that in both versions it is positive, as required in order to meet the conditions formulated in (2.3), and large, but insignificant. A possible explanation for the insignificance of the estimate of the parameter β could be the difference between consumption and expenditures, touched upon in the previous subsection. In order to determine whether this is the case, compare, for both verions, the results reported in column I with those reported in columns II and III. These two columns correspond with the two alternative assumptions regarding the link between consumption and expenditures introduced in the previous subsection.

Before turning to the estimate of β itself, notice that for the basic version of the model the other results reported in these columns are rather unaffected by the choice of the link between consumption and expenditures. For the household specific version the differences are somewhat larger. In household specific version II the estimate of e_1 is negative (but insignificant), implying a bliss point for the vacation good which (slightly) decreases with family size. This counter-intuitive result is present for both goods in version III, since both d_1 and e_1 are negative. The changes in the data set resulting from imposing the third assumption regarding the link between consumption and expenditures have another consequence, namely the rejection of the household specific version of the model by Hansen and Singleton's (1982) misspecification test.

Returning to the estimate of the parameter β, Tables 6 and 7 show that it is insignificant in all cases. So, it must be concluded that the transformation put forward in this chapter does not constitute a statistically significant element for the explanation of the pattern present in the original data set. Nor is this the case for the two data sets which result after imposing two, rather simple, alternative assumptions regarding the link between consumption and expenditures.

This finding is supported by another implication of the estimation results reported in Tables 6 and 7. Given the estimates of the model incorporating the transformation $g(\cdot)$, it is possible to determine whether an observation is located on the non-convex part of an indifference curve. Such an observation would in the transformed model imply non-optimizing behaviour on the part of the particular consumer.[13] In order to determine whether this occurs frequently, condition (2.2) can be used to calculate for each observation with positive vacation expenditures the inadmissable interval of vacation expenditures (given the reported non-vacation expenditures).

The percentage of observations with positive vacation expenditures which are correctly situated according to this criterion, i.e., which have vacation expenditures larger than the corresponding $\bar{y}_{h,\tau}$, are reported in Tables 6 and 7. Furthermore, the average minimum vacation expenditures required in order to be located on the convex part of the indifference curve, i.e., $\bar{y}_{h,\tau}$ averaged over months as well as over households, are also reported. From the tables it can be seen that both the percentage and the average minimum vacation expenditures are fairly insensitive to the chosen assumption regarding the link between consumption and expenditures. This is especially true for the household specific version. More importantly, the percentages reported are rather small.

Given the framework employed here, one could argue that the presence of intratemporal uncertainty causes a number of observations to be situated on the non-optimal part of the indifference curve. However, it seems unlikely that the presence of intratemporal uncertainty fully accounts for this outcome. It seems more probable that these small percentages are another indication for the fact that the proposed modification does not contribute considerably to the explanation of the variation present in the different data sets used in this section.

Table 6. Estimation results basic versions[1)]

	standard	I	II	III
b	-0.771 (0.163)	-0.497 (0.164)	-0.443 (0.163)	-0.528 (0.289)
c	-1.856 (0.102)	-3.326 (0.250)	-2.446 (0.381)	-2.129 (0.385)
d_0	87.04 (24.30)	19.09 (3.965)	26.58 (4.589)	38.22 (6.831)
e_0	93.59 (24.42)	54.25 (6.014)	46.67 (4.979)	72.30 (9.815)
β	.	252.0 (232.6)	245.1 (215.8)	2481 (1702)
$\frac{1+r}{1+\rho}$	0.999 (0.009)	1.000 (0.004)	0.999 (0.026)	1.000 (0.005)
T1	31.5	23.1	15.8	20.9
df1	17	16	16	16
p1	0.017	0.111	0.467	0.182
blx	97.9	82.0	89.0	88.3
bly	97.3	96.5	96.5	98.6
%	.	33.6	45.6	37.5
$\bar{y}$	.	5.15	5.40	7.72

1) consumption measured in hundreds of guilders
standard errors in parentheses
standard = model without transformation (cf. Table 2 in chapter 2)
I = model with transformation (3.1.4)
II = I, but with vacation expenditures smaller than Dfl. 100.- set equal to zero
III = I, but with three-monthly sum of vacation expenditures
T1 = chi-square value for Hansen and Singleton's misspecification test
df1 = degrees of freedom of misspecification test
p1 = significance level of misspecification test
blx = percentage of non-vacation expenditures satisfying the bliss point condition
bly = percentage of vacation expenditures satisfying the bliss point condition

% = percentage of observations (with positive vacation expenditures) situated on the convex part of an indifference curve

$\bar{y}$ = average vacation expenditures at which the point of inflexion of the indifference curves is located (in hundreds of guilders)

Table 7. Estimation results household specific versions[1)]

	standard	I	II	III
b	-0.523 (0.182)	-0.732 (0.290)	-0.451 (0.204)	-0.132 (0.310)
c	-1.660 (0.069)	-3.022 (0.323)	-2.271 (0.262)	-2.420 (0.412)
d_0	88.61 (28.11)	18.27 (4.272)	23.85 (5.168)	16.74 (3.135)
d_1	31.32 (13.57)	2.562 (1.353)	1.019 (0.986)	-3.149 (0.554)
e_0	94.41 (28.61)	52.39 (9.125)	42.25 (5.667)	48.01 (6.074)
e_1	27.62 (14.04)	1.779 (3.047)	-1.137 (2.072)	-15.80 (4.445)
β	.	234.1 (247.7)	162.2 (143.9)	703.0 (2273)
$\frac{1+r}{1+\rho}$	1.000 (0.003)	0.999 (0.028)	0.999 (0.026)	0.990 (0.211)
T1	18.8	14.5	16.5	37.4
df1	15	14	14	14
p1	0.222	0.413	0.284	0.001
blx	99.1	53.3	83.9	67.9
bly	99.6	97.0	96.4	96.8
%	.	35.2	35.1	34.4
$\bar{y}$	.	4.91	4.90	5.15

1) see the legend of Table 6

4. Summary and conclusions.

In this chapter it was investigated whether a consumption pattern in which no low consumption levels are present, can be explained within a life cycle context. Since neither the standard life cycle model, nor some straightforward extensions turned out to be fully suited for explaining such a consumption pattern, an alternative was proposed. It consisted of introducing a transformation in either the utility function or the budget constraint, which was chosen such that either the preference ordering or the budget constraint was not convex for small values of the good displaying the aforementioned consumption pattern.

An example of such a modified life cycle model was estimated, using a panel containing, among other variables, the monthly expenditures on vacation and non-vacation. Under different assumptions regarding the link between consumption and expenditures, a quadratic utility function in which the vacation good was replaced by a tranformation was estimated. The estimation results indicated that under none of the assumptions regarding the link between consumption and expenditures did the proposed transformation contribute significantly to the explanation of the data.

In order to determine whether transforming the life cycle model in the way put forward in this chapter is in general unwarranted, further research is needed. Apart from the usual directions this research could take, like trying an alternative specification of the transformation, or a more general specification of the life cycle model (for example, including a seasonal effect, interdependent preferences, or institutional constraints to explain why most people go on vacation when it is most expensive, as noted in section 2), there are some interesting alternatives.

One could, for example, apply the modified life cycle model introduced in this chapter in labour supply studies. This might be interesting, since the observation problems in case of labour supply are likely to be smaller than in case of disaggregated consumption considered here.

Alternatively, one could investigate the link between consumption and expenditures more thoroughly than the rather ad hoc set up employed in this chapter. This latter topic will be taken up in the next chapter.

APPENDIX A.

In this appendix, the conditions under which the modified life cycle model has a solution which can be characterized by the first order conditions are determined. That is, the conditions under which the usual estimation approach, in which the first order conditions are used, can be applied.

The existence of a solution is ensured since the conditions imposed by Melenberg and Alessie (1989) guaranteeing this, i.e., the continuity of the objective function and the compactness of the choice set, also hold here.

Turning to the second aspect, model (2.4) and its solution satisfy the conditions under which the generalized Lagrange multiplier rule as given by Melenberg and Alessie (1989) can be applied. However, since they formulate the multiplier rule in quite general terms, two additional assumptions are imposed by them which are suficient to make it suited for empirical applications.

The first one is that the solution must be an internal point of the domain of the consumption functions. Since at the optimum the nonnegativity constraints can be binding in model (2.4), the domain must be chosen in a way that ensures that consumption paths with zero consumption of a commodity in one or more periods are internal points. Such a domain is defined in chapter 2.

The second condition is a normalization condition. However, as it assumes a concave lifetime utility function, it cannot be used for model (2.4). Instead, a condition given by Luenberger (1969, pp. 248-249) is imposed, namely that the solution is a regular point. This condition essentially requires that the choice set has at least one internal point. The choice set of model (2.4) is larger than the one corresponding to the standard model, as it includes zero consumption of the different commodities. Since the choice set of the standard model has a nonempty interior (cf. Melenberg and Alessie (1989)), the requirement for model (2.4) is met.

Appendix B.

Under the assumption that consumers decide on vacation before deciding on the non-vacation good, the cross section used by Van Soest and Kooreman (1987) allows for the estimation of the parameters of interest on the basis of the following intratemporal moment restriction:[14]

$$E_t[(\frac{\partial u(x_t,g(y_t))}{\partial x_t} \cdot \frac{p_{t,y}}{p_{t,x}} - \frac{\partial u(x_t,g(y_t))}{\partial y_t}) \cdot I_{(0,\infty)}(y_t)] = 0 \quad (B.1)$$

where the indicator function is needed to eliminate the Lagrange multiplier corresponding with the nonnegativity constraint for the vacation good (see chapter 2 for details). The presence of the indicator function implies that all households reporting zero expenditures on vacation are not taken into account when estimating this system. By multiplying this conditional moment restriction by a vector of (properly chosen) instruments, a system of unconditional moment restrictions can be obtained. Using the specification given in section 3.1 of this chapter, this system can be used in estimation.

It turns out that the estimate of the parameter β of the transformation given in (3.1.4) is close to zero (0.0004), and insignificant (the corresponding standard error is 0.08). This outcome implies that the proposed transformation does not have a significant impact. This result can be seen as supporting the view that the transformation is superfluous, or that the assumption regarding the ordering of the consumption decisions is inappropriate.

However, the following explanation might also be valid. The transformation was introduced to explain that for a given household the consumption of a certain good fluctuated strongly over time. The data set used, however, just covers a single period, implying that the jump in consumption levels the transformation was intended to explain, is not present in the data. So, the insignificance of the transformation can also be interpreted as an indication that data on more than one period is required to enable an assessment of the role of the proposed modification.

Notes to chapter 3

1 By this is meant that the observation period is relatively short, e.g. a month for the data set used in this paper.

2 Note that studies in this field usually do not work within a life cycle framework.

3 For those households reporting more than one vacation, only the corresponding average vacation expenditures can be determined. This is the case for about thirty per cent of the vacation expenditures. Excluding these observations from the data set leaves the distribution as reported in Table 1 essentially unchanged.

4 It will be assumed throughout this study, that good x is consumed in each period.

5 In contrast with the number of households reporting vacation expenditures, the average monthly vacation expenditures (which are obtained by averaging over the positive vacation expenditures in a month), although varying over months, do not exhibit a clear seasonal pattern.

6 This framework also allows for the incorporation of the aforementioned seasonal and intertemporal aspects, as well as other elements (like interdependent preferences). However, as this study wants to focus on the jump in the consumption level, these aspects are not considered here.

7 The utility function $u(x_\tau, g(y_\tau))$ is still strictly concave with respect to x_t and $g(.)$.

8 For notational convenience, the period index is suppressed.

9 Strict concavity implies that the matrix of second order derivatives of $u(\cdot)$ is negative definite. This in turn implies that the matrix in

which the diagonal elements are multiplied by minus one is positive definite. Hence the first term between square brackets in equation (2.2) is positive.

10 For the following two reasons, introducing fixed costs in model (2.1) will not be considered. Firstly, because the presence of fixed costs implies non-differentialbility at zero, the generalized Lagrange multiplier rule used in this study for deriving the first order conditions can not be applied (see, for example, Melenberg and Alessie (1989) for conditions under which this rule can be applied). Secondly, the usual way of solving a fixed costs model, i.e. comparing the utility levels of all commodity bundles satisfying the first order conditions, is less suited in a life cycle setting. This because it involves comparing the expected utility of all lifetime consumption paths satisfying the first order conditions. In order to be able to do this, information on matters like the lifetime and the distribution of the uncertainty inducing variables is needed. Since this information is not available, and this study wants to do without assumptions regarding these matters, the above described procedure can not be used (see Rust (1987) for an example, albeit in a somewhat different context, of this approach, if one is willing to make such assumptions).

11 The assumption that the consumption of the non-vacation good is approximately equal to the reported expenditures is maintained.

12 Alternatively, vacation outlays below Dfl. 100.- could be added to the non-vacation expenditures. Given that these amounts are rather small, and the fact that most observations are not changed by either approach, it is unlikely that this alternative would lead to very different estimation results.

13 Notice that this implication does not hold in the standard life cycle model, since small values can be optimal in this model.

14 The assumption on the ordering of consumption decisions is required in order to be able to derive restriction (B.1) from the first order conditions. That is, this restriction corresponds with a choice of the functions h_{tx} and h_{ty} (cf. chapter 2 for details on these functions) which makes it necessary that each consumer knows both the price and the consumption level of y_t, before deciding on x_t.

CHAPTER 4

EXPENDITURE VERSUS CONSUMPTION IN THE MULTI-GOOD LIFE CYCLE CONSUMPTION MODEL

1. Introduction.

In empirical studies on consumer behaviour, there is often a contrast between the theoretical and empirical sections. The models formulated in the theoretical part are concerned with consumption. The data sets used in the empirical part, however, rarely contain information on the actual consumption of consumers. Instead, they usually contain information on the purchases made by consumers. Depending on, among other factors, the extent to which the data are disaggregated into different commodity categories, and the length of the time interval used as reporting period, the distinction between consumption and expenditures may be important. Ignoring this distinction can lead to incorrect inferences on the performance of the model under consideration.

Because of this potential problem, some attempts have been made to take the aforementioned difference into account in the modelling phase. In the next section some approaches suggested in the literature are discussed. As they all suffer from some theoretical drawbacks, section 3 is devoted to the development of a model that tries to improve on these existing alternatives, assuming a life cycle context. In section 4, the empirical applicability of this modified life cycle model is investigated. Attention is focussed especially on the situation which is most likely to occur, i.e., in which one has information on purchases, but not on consumption. It turns out that this lack of information seriously limits the applicability of the model. More precisely, given this data limitation, the objective function of the modified model will generally not retain its (expected) utility function format. If one assumes perfect foresight on the part of the consumer, the objective function does retain its original format. Finally, some concluding remarks are made in section 5.

2. Existing solutions.

Several ways of modelling the difference between consumption and expenditures have been proposed in the literature. The best-known of these approaches probably is the one suggested in the so-called 'infrequency of purchase' literature, which emerged from studying consumption on a disaggregated level.

The starting-point for this model is often a demand equation for a certain good, where the fact that some persons are found not to consume the good, is usually dealt with by assuming a Tobit specification for this equation (cf., for example, Deaton and Irish (1984), Blundell and Meghir (1987) and Pudney (1989), section 4.4). Next, a link between (unobserved) consumption and (observed) expenditures is established by taking into account that what is bought during the reporting period, is not necessarily also consumed in the same period. This difference implies that the expenditure data are likely to differ from the underlying consumption pattern in the following two ways: firstly, the number of individuals reporting zero expenditures on a good will be larger than the number of individuals not consuming the commodity, and secondly, if the expenditures reported by individuals are positive, they will, on average, be larger than the corresponding consumption of these individuals. The infrequency of purchase approach tries to correct for these possible differences by adding an additional censoring process to the Tobit specification, thus scaling the positive expenditures downwards, and allowing for zero expenditures while the underlying consumption is positive. There are several objections which could be raised against this approach.

An important drawback of this approach is that the model is static. That is, although the infrequency of purchase model tries to establish a link between consumption and expenditures, it only links these quantities on a period by period basis. But because it is often possible that consumption in a certain period can be paid for not only in the period itself but also in periods preceding or following it, the link between consumption and expenditure should preferably be a multi-period one.[1] Ignoring this intertemporal aspect can easily result in incorrect inferences on the consumption pattern of consumers. To illustrate this, consider the following three-period example: assume someone consumes a certain good only in the second period, but divides the payment for it over all three periods. If this difference between the consumption and expenditure patterns is systematic[2], the infrequency of purchase models put forward in the literature will predict a consumption level for the second period which is at most equal to the purchases made in that period, and will, with positive probability, predict a positive consumption level in the other two periods. So, trying to model a dynamic process in a static framework can lead to serious distortions.

In order to take account of this problem, one could try to incorporate the infrequency of purchase approach in a dynamic model. As was pointed out already in chapter 1, in this study attention will be restricted to probably the best-known dynamic model: the life cycle model. Some of the consumption functions used in the static models can be interpreted as resulting from the second stage of a life cycle model in two-stage budgeting form (see, for instance, Meghir and Robin (1989)). So, a straightforward way of introducing dynamics could be to alter the life cycle model in such a way that the resulting consumption equations are no longer static, but also depend on past and future consumption. In this way one obtains a relation between consumption over time and thereby, after substituting the corresponding links between consumption and expenditures, in a relation between the expenditures in different periods. However, these links themselves remain static, implying that such a model still suffers from the aforementioned drawback.

Therefore, an alternative way of linking consumption and expenditures in a life cycle context will be proposed, which will be formalized in the next section. It can briefly be described as follows: the model retains the notion present in some infrequency of purchase studies, notably Meghir and Robin (1989), that individuals decide on their consumption and purchase strategies simultaneously. But in contrast with these studies, it takes account of this simultaneity by incorporating the link between consumption and expenditures from the outset in the life cycle model. Hence, the link is an integral part of the life cycle model itself, and no longer a separate model, employed only after the life cycle model has been solved.

The life cycle model which thus results has the following main characteristics: since the consumer derives utility from consuming a good and not from buying it, the utility function has consumption variables as its arguments. For the budget constraint, however, the opposite is true, that is, the price when buying a good is essential, not when consuming it. Therefore, the budget constraint depends on quantities bought, and not on quantities consumed. The link between the two is established by the fact that what is consumed in a certain period must be paid for sometime during the lifetime. This link implies, for instance, that aspects determining the expenditure pattern, like the timing of payments, can also influence consumption. So, the choice of the consumption path and the choice of the expenditure pattern determine, either directly or indirectly, the maximum expected utility which can be obtained by a consumer.

Apart from the aforementioned fact that the infrequency of purchase models suggested in the literature are static, they have another, less important, drawback. In order to be able to employ the usual estimation framework, i.e., some sort of Tobit specification, it often is assumed that the consumption and expenditure variables are normally distributed. This normality assumption could be a source of misspecification. Since one of the advantages of the life cycle model is that it can be estimated without imposing such distributional assumptions, one would rather do without them.[3]

The remainder of this section is devoted to a discussion of two other ways of linking consumption and expenditures which are proposed in the literature, and which can be considered as special cases of the model to be introduced in the next section. The two links can be regarded as two different representations of one and the same model, which will be called the 'lag' model. It allows for an intertemporal link between consumption and expenditures, and is usually applied within a life cycle context. Starting-point for this lag model is the assumption that because of the durability of commodities, consumption in a certain period can be paid for in that period, or in previous periods. This is modelled by equating the consumption in a period to a function of the purchases made in that period and in earlier periods.

The first representation links current consumption to current and past expenditures by means of a lag polynomial. In some studies the polynomial is assumed to be finite (see e.g. Hansen and Singleton (1983), Hayashi (1985b), Muelbauer (1988), Eichenbaum, Hansen and Singleton (1988) and Dunn and Singleton (1986)), whereas it is assumed infinite in other studies (cf., for instance, Neusser (1988) and Dunn and Singleton (1986) for durables).

The second, nowadays less commonly used representation, links consumption and expenditures by introducing a stock model. Purchases of a good lead to an increase of the available stock of this good, and the assumption that a constant fraction of this stock is consumed in every period establishes the link between consumption and expenditures.[4] Examples of this approach are the papers by Spinnewyn (1981), Pashardes (1986) and Bar-Ilan and Blinder (1988).

The equivalence of the two representations is easily demonstrated. Consider, for instance, the following version of the former representation (where $N \le \infty$):

$$c_{t,i} = A(L)e_{t,i} \equiv \textstyle\sum_{j=0}^{N} a_{j+1,i} \cdot e_{t-j,i}; \qquad 0 \le a_{j+1,i} \le 1 \qquad (2.1)$$

where $c_{t,i}$ = period t's consumption of good i

$e_{\tau,i}$ = period τ's expenditutres on good i, $\tau = t-N,\dots,t$

A similar version of the second representation can be written as follows:

$$\begin{aligned} s_{t,i} &= \textstyle\sum_{j=0}^{N} \vartheta_{j+1,i} \cdot e_{t-j,i}; \qquad 0 \le \vartheta_{j+1,i} \le 1 \\ c_{t,i} &= \alpha_i \cdot s_{t,i}; \qquad 0 \le \alpha_i \le 1 \end{aligned} \qquad (2.2)$$

where $s_{t,i}$ = period t's stock of good i

By choosing $a_{j+1,i}$ equal to $\alpha_i \cdot \vartheta_{j+1,i}$, the equivalence of both representations is established.

As stated before, the usual reason for introducing either one of these representations into a model constructed for explaining consumer behaviour, is to capture the difference between consumption and expenditures resulting from the durability aspect of some goods. However, if consumption is sufficiently disaggregated both over goods and over time,[5] there are also other reasons causing consumption to differ from expenditures. The most important one is the timing of the (registration of) payments. Some goods must be paid in advance (for example holiday reservations), whereas others can be paid after they have been consumed (for instance the telephone bill).[6] Moreover, even if the goods are paid during the period they are consumed, the payments need not be (completely) reported in this period. For example, it takes some time before payments made abroad are processed by banks and brought to one's attention. Or one could buy goods using a credit-card, which are charged only weeks later. If the reporting period is short, for example two weeks like in the often used British Family Expenditure Surveys, consumption and reported expenditures can differ, depending on what information is used by consumers when reporting their expenditures. Since these differences between the consumption and expenditure patterns are not taken into account by the lag model, it is less suited for modelling consumer behaviour on a disaggregated level.

Another reason for making the lag model less appropriate for modelling consumer behaviour at such a disaggregated level, is that it is difficult to account for zero consumption in this framework. This is because, as mentioned before, in many studies using this approach, it is assumed that the lag polynomial has an infinite length. This implies, for instance, that in case of the often used model employing a geometric decay structure, that once a purchase is made, the model "predicts" a positive consumption level (however small) in the period the purchase is made, and in all subsequent periods, thus resulting in a consumption path which is likely to be too smooth. Some studies try to overcome this by imposing some maximimum lag (typically one or two periods). However, since the reasons for choosing this maximimum lag are usually data driven rather than resulting from theoretical considerations, this is also not fully satisfactory. As zero consumption is likely to occur frequently if the consumption is sufficiently disaggregated, the above indicates that the lag model is less suited for handling such problems.

A less important disadvantage of the lag model is that it is difficult to combine with habit formation. Introducing habits in the lag polynomial representation requires incorporating an additional lag polynomial linking consumption over time.[7] However, the lag structure resulting from combining the two lag polynomials is by no means uniquely related to one particular combination of polynomials, as is illustrated by the following example.

Assume the following link between total consumption (c) and total expenditures (e):

$$c_t = a \cdot e_t + b \cdot e_{t-1} \tag{2.3}$$

Furthermore assume that the negative effect of yesterday's consumption on utility derived from today's consumption, caused by habit formation, is as follows (where c_t^* is what Spinneweyn (1981) called uncommited consumption):

$$c_t^* = c_t - d \cdot c_{t-1} \tag{2.4}$$

Under the assumption that one has information on expenditures but not on consumption, one needs to combine both equations in order to obtain an expression for c_t^* in terms of the observed variables:

$$c_t^* = a \cdot e_t + (b - d \cdot a) \cdot e_{t-1} - d \cdot b e_{t-2} = \alpha \cdot e_t + \beta \cdot e_{t-1} + \gamma \cdot e_{t-2} \tag{2.5}$$

The lag structure presented in (2.5), resulting from equations (2.3) and (2.4), can also be obtained by assuming, for example, no habit formation and a two-period lag for the link between consumption and expenditures, or assuming that expenditures equal consumption and that habits have an influence lasting two periods. The fact that such a one-to-one correspondence is lacking, implies that one cannot draw any clear-cut conclusions on the importance of habit formation on the one hand, and the link between consumption and expenditures on the other.

In the studies using the stock representation it is usually possible to draw conclusions on the role which each aspect plays. For instance, if one assumes, like Pashardes (1986), that the link between consumption and stock is linear and constant over time, and that the maximum lag (N in equation (2.2)) is infinite, the sign of the parameters $\vartheta_{j,i}$ in equation (2.2) determines whether the habit forming aspect outweighs the durability aspect. However, if not all of these assumptions are satisfied, this needs no longer hold true. If, for example, the link between consumption and stock is nonlinear, such an unambiguous interpretation of the parameters $\vartheta_{j,i}$ probably will not be possible.

Especially the assumption that a constant fraction of the stock is consumed in each period is troublesome, since the size of the stock is partly determined by market factors (like prices, and the minimum quantity of a good one must buy), whereas consumption is mainly determined by preferences. Assuming that a constant fraction of the stock is consumed implies, for example, that if a price cut in a certain period induces a consumer to buy a large quantity of a good to take advantage of this discount, his or her consumption *must* increase significantly in this period. Since there is no compelling reason why consumers should behave so rigidly, alternative consumption patterns could be just as plausible. One such alternative could be a pattern implying a constant consumption level as long as the available stock allows for it; so $c_t = \min[\bar{c}, s_t]$. Such a pattern perhaps could be a reasonable representation of the consumption of a commodity like clothing.

Given the aforementioned shortcomings, the lag model is not considered fully suited for establishing a link between consumption and expenditures, when studying individual consumer behaviour on a disaggregated level. The next section is, therefore, devoted to the development of an alternative framework which tries to improve upon the alternatives discussed in this section.

3. Linking consumption and expenditures in the life cycle model.

The models discussed in the previous section can be considered as belonging to either one of two different classes of life cycle models. The classes differ in the way in which they take account of the distinction between expenditures and consumption. The first class contains the infrequency of purchase models which can, under appropriate assumptions, be interpreted as a framework in which one first solves a life cycle model formulated in consumption terms only. In a second step, the thus resulting consumption equations are related to some expenditure variable.

In contrast, the second class of life cycle models, to which the lag model belongs, is characterized by the fact that the difference between consumption and expenditures is taken into account in the life cycle model itself. That is, the link between the consumption and expenditure variables is introduced by adding equality constraints to the model which link each period's consumption to some function of realized purchases. Because of this exact relationship between the consumption and purchase levels, choosing either of them, fully determines the other.

In this section a generalization of the models belonging to this second class is put forward. The key notion underlying these models- that utility is derived from consumption, and costs result from purchases- is retained. Hence, the utility function depends on consumption variables, and the budget constraint is determined by expenditure variables. The difference between the proposed model and the lag model is the way in which consumption and expenditures are linked to one another.

It is no longer assumed that the consumption in a certain period is exactly equal to a weighted sum of purchases realized until that period. Instead, it is assumed that expenditures imply an upper bound on the consumption of the different goods. If a particular good is a durable, the corresponding upper bound will depend on past purchases and past consumption. If one can postpone the payment of the consumption of a good, the corresponding upper bound will depend on the purchases which will be made in future periods.[8]

An advantage of the model proposed in this section, as compared with the lag model, is that it allows for a greater flexibility. For example, a price discount in a certain period might induce a consumer to buy a large quantity of the particular good in that period, without increasing his consumption of the good.[9] Or one might buy a durable good, for instance a car, whilst keeping one's consumption of the good unchanged. Because of the assumed equality between consumption and a weighted sum of realized purchases, both cases are not easily modelled using the lag model. In the framework proposed in this section, however, they can be modelled without great difficulty, since both examples simply lead to higher upper bounds for the particular goods. Thus higher consumption levels of these goods are possible, but by no means necessary.

A formal representation of the life cycle model in which this generalization is incorporated could be as follows (for $t = 1,\ldots,L$):[10]

$$\max_{c_t, e_t, \ldots c_L, e_L} \; E_t \sum_{\tau=t}^{L} \left(\frac{1}{1+\rho}\right)^{\tau-t} u(c_\tau)$$

$$\text{s.t. } \sum_{\tau=t}^{L} \left(\frac{1}{1+r}\right)^{\tau-t} p'_\tau e_\tau \;\le\; (1+r)A_{t-1} + \sum_{\tau=t}^{L} \left(\frac{1}{1+r}\right)^{\tau-t} i_\tau,$$

$$c_\tau \ge 0 \quad \tau = 1,\ldots, L,$$

$$c_1 \le \delta_{1,1} e_1 + \Sigma_{s=2}^{L} \alpha_{s,1} e_s, \qquad (3.1)$$

$$c_\tau \le \delta_{\tau,\tau} e_\tau + \Sigma_{s=1}^{\tau-1} \delta_{s,\tau}(e_s - c_s) + \Sigma_{s=\tau+1}^{L} \alpha_{s,\tau} e_s \quad \tau = 2,\ldots, L-1,$$

$$c_L \le \delta_{L,L} e_L + \Sigma_{s=1}^{L-1} \delta_{s,L}(e_s - c_s).$$

where

$u(\cdot)$ = within period utility function; assumed to be strictly concave, constant over time and increasing in its arguments,

c_τ = $(c_{\tau,1},\ldots,c_{\tau,M})'$: M-dimensional vector of *consumption* of goods in period τ,

e_τ = $(e_{\tau,1},\ldots,e_{\tau,M})'$: M-dimensional vector of *purchases* of goods in period τ,

p_τ = $(p_{\tau,1},\ldots,p_{\tau,M})'$: M-dimensional price vector of the goods in period τ,

i_τ = nominal non-property income in period τ,

r = nominal interest rate; assumed to be constant over time,

ρ = time preference parameter,

A_{t-1} = assets available at the beginning of period t,

E_t = expectation conditional on the information available at period t,

$\alpha_{s,\tau}$ = $\mathrm{diag}(\alpha_{s,\tau,1},\ldots,\alpha_{s,\tau,M})$; (M×M) diagonal matrix with as diagonal elements the fractions of period s' expenditures on each good which can be consumed in period $\tau \le s$,

$$\begin{aligned} &\alpha_{s,\tau,i} \in [0,1] \\ & \qquad\qquad i = 1,..,M; \quad \tau = 1,..,L; \quad s = \tau+1,..,L \\ &\alpha_{s,\tau+1,i} \ge \alpha_{s,\tau,i} \end{aligned} \tag{3.2}$$

$\delta_{s,\tau}$ = $\mathrm{diag}(\delta_{s,\tau,1},\ldots,\delta_{s,\tau,M})$; (M×M) diagonal matrix with on the diagonal the fractions of not consumed outlays on each good done in period s, which are available in period $\tau \ge s$,

$$\begin{aligned} &\delta_{1,\tau,i} = 1 \text{ if } c_{1,i} \ge e_{1,i} \\ & \qquad\qquad i = 1,..,M; \; \tau = 1,...,L \\ &\delta_{1,\tau,i} \in [0,1] \text{ if } c_{1,i} < e_{1,i} \end{aligned}$$

$$\begin{aligned} &\delta_{s,\tau,i} = 1 \text{ if } c_{s,i} \ge e_{s,i} + \Sigma_{l=1}^{s-1}\delta_{l,s,i}(e_{l,i} - c_{l,i}) \\ & \qquad\qquad i = 1,..,M; \; s = 2,..,L; \; \tau = s,..,L \\ &\delta_{s,\tau,i} \in [0,1] \text{ if } c_{s,i} < e_{s,i} + \Sigma_{l=1}^{s-1}\delta_{l,s,i}(e_{l,i} - c_{l,i}). \end{aligned}$$

As was the case with the models introduced in the previous two chapters, the model put forward here again is a modification of the multi-good version of Hall's (1978) life cycle consumption model under uncertainty. As can be seen from the above formulation, a consequence of loosening the tie between consumption and expenditures is that in order to achieve the maximum expected lifetime utility, the model must be solved with respect to the consumption as well as the expenditure variables. This contrasts with the lag model, in which the link between expenditures and consumption implies that the models need to be maximized only with respect to either one of these variables. As stated before, the difference between consumption and expenditures implies a budget constraint which depends on expenditure variables. The other constraints in (3.1) provide the link between consumption and purchases.[11] Their specific form is determined by the aspects mentioned earlier: durability and postponement.

If a good is durable, a certain fraction of the quantity bought in a period will, by definition, be available in the next period(s). This aspect is represented by the part of the right hand side of these constraints relating to past expenditures. As can be seen from the way in which this is modelled, both the consumption in the periods prior to the particular period under consideration, as well as technical factors influencing the rate of decay ($\delta_{s,\tau}$), determine the exact quantity which is available in future periods.

Notice that in lag models these two elements are usually not separated. Especially the effect of consuming on the stock available next period is ignored, as depreciation is considered to be the only reason for a decrease in the available stock.

The second aspect determining the link between consumption and expenditures is the postponement of payments. This implies that a certain quantity of a good can be consumed in one period, and only be paid for in later periods. This aspect is represented by the part of the right hand side of the 'linking' constraints relating to purchases in periods succeeding the particular period under consideration. The assumption that payments which can be delayed $s-\tau$ periods can also be postponed one period less, implies the restriction on the $\alpha_{s,\tau}$ given in (3.2).

Another effect worth pointing out is that incorporating the postponement aspect complicates the way in which the durability aspect is modelled. The possibility of delaying the payment implies that one has to determine for each good in each period, starting with period t, whether the corresponding consumption level exceeds the quantity remaining from the purchases made until that moment. If this is the case, a certain quantity has to be paid for in later periods, hence the durability parameter corresponding to this good and period, i.e., $\delta_{s,\tau,i}$, is set equal to one, since future payment obligations do not decrease over time. If the consumption level does not exceed the available quantity, the durability parameter takes a value between zero and one, depending on technical factors influencing the rate of decay. Hence, each $\delta_{s,\tau,i}$ is actually a function of consumption and expenditure variables.[12] In order to be able to apply the generalized Lagrange multiplier rule used in this study, it is necessary to assume that these functions are differentiable.

Finally, notice that nonnegativity constraints are imposed on the consumption variables only. Expenditures can become negative, since one can sell (a part of) the stock one has built up in previous periods.[13] So the lower bound for the purchases of a good in a certain period is minus the quantity available at the beginning of this period.

In the remainder of this section it will be established that the life cycle model without the difference between consumption and expenditures, the lag model, and the life cycle model with habit formation, all are special cases of the model given in (3.1). The common features of these models are that they do not allow for the postponement of payments, and that the quantity which is available for consumption in each period is also consumed in that period. This implies that in the constraints linking consumption to expenditures the $\alpha_{s,\tau}$ are set equal to zero, and that the inequality signs are replaced by equality signs.

The additional requirement needed in order to obtain the traditional life cycle model in which the difference between consumption and expenditures is not taken into account, is to set all $\delta_{s,\tau}$, $s \neq \tau$ equal to zero and all $\delta_{\tau,\tau}$ equal to one. The lag model results if one imposes a reparameterization on the $\delta_{s,\tau}$. Under the assumption that $\vartheta_{i,j+1} = \vartheta_i^{j+1}$, i.e., the familiar geometric decay structure, the stock representation as given in (2.2), for instance, results if one substitutes the consumption realized until period τ in period τ's constraint, and makes use of the following reparameterization (with $N = \tau-1$):[14]

$$\delta_{s,\tau,i} = \alpha_i \vartheta_i \left(\frac{\vartheta_i}{1-\alpha_i\vartheta_i}\right)^{\tau-s} \qquad i = 1,..,M;\ s = 1,..,\tau;\ \tau = 1,..,L$$

Similarly, under the assumption that $a_{i,j+1} = a_i^{j+1}$, the polynomial representation given in (2.1) results after substituting in each period's constraint the consumption realized until that period, and using the following reparameterization:

$$\delta_{s,\tau,i} = a_i\left(\frac{a_i}{1-a_i}\right)^{\tau-s} \qquad i = 1,..,M;\ s = 1,..,\tau;\ \tau = 1,..,L$$

Since habit formation is modelled by introducing a similar polynomial, as was already pointed out in section 2, the life cycle model with habit formation can be obtained using the same procedure.

So it can be concluded that a number of well-known models are special cases of the life cycle model introduced in this section. In the next section it is determined under what conditions this life cycle model can be used in empirical applications.

4. Empirical applicability.

The life cycle model as formulated in (3.1) can be estimated if one has information on both consumption and expenditures. However, as already mentioned before, it is very rare to find a data set containing information on consumption.[15] In most data sets one only finds information regarding the purchase of commodities. Hence, the question which is addressed first, is what conditions must be imposed to enable the estimation of the model given in (3.1) using expenditure data only. In subsection 4.1, this question is taken up for the model under uncertainty, whereas the life cycle model assuming perfect foresight is considered in subsection 4.2. In subsection 4.3 the conclusions for the life cycle model under uncertainty arrived at in subsection 4.1 are compared with those which can be drawn if one has consumption data at one's disposal.

4.1 The life cycle model under uncertainty.

The usual way of estimating models like the one given in (3.1) is to combine the first order conditions into a system of equations which can be estimated on the basis of the data available. In order to derive the first order conditions for the model given in (3.1), the same method is applied as in the previous two chapters, i.e., a generalized Lagrange multiplier rule. Similar to the way in which the first order conditions were derived for the models considered in chapter 2, the first order conditions for model (3.1) can be obtained:

$$E_t[\ \sum_{\tau=t}^{L}(\frac{1}{1+\rho})^{\tau-t}D_c u(c_\tau)'h_\tau^c - \lambda_t\cdot\sum_{\tau=t}^{L}(\frac{1}{1+r})^{\tau-t}p_\tau' h_\tau^e + \sum_{\tau=1}^{L}\mu_\tau' h_\tau^c +$$

$$\sum_{\tau=1}^{L}\sum_{i=1}^{M}\nu_{\tau,i}\cdot[D_e R_{\tau,i}' h_i^e - D_c R_{\tau,i}' h_i^c]\ ] = 0 \qquad (4.1)$$

such that

$$E_t\ [\mu_{\tau i}\cdot c_{\tau i}] = 0,$$

$$i = 1,\ldots, M;\ \tau = 1,\ldots, L$$

$$E_t\ [\nu_{\tau i}\cdot R_{\tau i}] = 0.$$

where

$D_c u(c_\tau) = (\frac{\partial u(c_\tau)}{\partial c_{\tau,1}},\ldots, \frac{\partial u(c_\tau)}{\partial c_{\tau,M}})'$: vector of partial derivatives of $u(\cdot)$ with respect to the consumption variables,

h_τ^c = $(h_{\tau,1}^c,\ldots, h_{\tau,M}^c)'$: vector of functions were $h_{\tau,i}^c$ is allowed to depend on all variables influencing $c_{\tau,i}$ (cf. the application of the generalized Lagrange multiplier rule in section 3 of chapter 2),

h_τ^e = $(h_{\tau,1}^e,\ldots, h_{\tau,M}^e)'$: vector of functions were $h_{\tau,i}^e$ is allowed to depend on all variables influencing $e_{\tau,i}$,

h_i^c = $(h_{1,i}^c,\ldots, h_{L,i}^c)'$,

h_i^e = $(h_{1,i}^e,\ldots, h_{L,i}^e)'$,

λ_t = the nonnegative Lagrange multiplier corresponding with the lifetime budget constraint,

μ_τ = $(\mu_{\tau,1},\ldots, \mu_{\tau,M})'$: vector of nonnegative Lagrange multipliers correpsonding with the nonnegativity constraints for period τ's consumption,

$\nu_{\tau,i}$ = nonnegative Lagrange multiplier corresponding with the upper bound on period τ's consumption of good i,

$R_{\tau,i}$ = the upper bound on period τ's consumption of good i, i.e., the last restriction in (3.1), with $c_{\tau,i}$ substracted from both sides of this quantity constraint,

$D_e R_{\tau,i}$ = $(D_{e_1} R_{\tau,i}, \ldots, D_{e_L} R_{\tau,i})'$: $R_{\tau,i}$ differentiated with respect to $e_{s,i}$, s = 1,..., L

$$D_{e_s} R_{\tau,i} = \delta_{s,\tau,i} + \sum_{l=s}^{\tau-1} \frac{\partial \delta_{l,\tau,i}}{\partial e_{s,i}} (e_{l,i} - c_{l,i}) + \frac{\partial \delta_{\tau,\tau,i}}{\partial e_{\tau,i}} e_{\tau,i} \quad s < \tau,$$

$$D_{e_\tau} R_{\tau,i} = \delta_{\tau,\tau,i} + \frac{\partial \delta_{\tau,\tau,i}}{\partial e_{\tau,i}} e_{\tau,i},$$

$$D_{e_s} R_{\tau,i} = \alpha_{s,\tau,i} \quad s > \tau,$$

$D_c R_{\tau,i}$ = $(D_{c_1} R_{\tau,i}, \ldots, D_{c_L} R_{\tau,i})'$: $R_{\tau,i}$ differentiated with respect to $c_{s,i}$, s = 1,..., L

$$D_{c_s} R_{\tau,i} = -\delta_{s,\tau,i} + \sum_{l=s}^{\tau-1} \frac{\partial \delta_{l,\tau,i}}{\partial c_{s,i}} (e_{l,i} - c_{l,i}) + \frac{\partial \delta_{\tau,\tau,i}}{\partial c_{\tau,i}} e_{\tau,i} \quad s < \tau,$$

$$D_{c_\tau} R_{\tau,i} = -1 + \frac{\partial \delta_{\tau,\tau,i}}{\partial c_{\tau,i}} e_{\tau,i},$$

$$D_{c_s} R_{\tau,i} = 0 \quad s > \tau.$$

In order to be able to estimate the first order conditions formulated above in the absence of consumption data, it will, in general, be required that the parts depending on (unobserved) consumption variables are eliminated. This is achieved by setting all $h^c_{\tau,i}$ equal to zero. After this choice of the $h^c_{\tau,i}$ is substituted in (4.1), the following condition results:

$$E_t[-\lambda_t \cdot \sum_{\tau=t}^{L}(\frac{1}{1+r})^{\tau-t} p_\tau' h_\tau^e + \sum_{\tau=1}^{L}\sum_{i=1}^{M} \nu_{\tau,i} \cdot D_e R_{\tau,i}' h_i^e] = 0 \quad (4.2)$$

As (4.2) demonstrates, this procedure does not allow for the estimation of the parameters of the utility function. Nor is it straightforward to estimate the parameters characterizing the link between consumption and expenditures, i.e., the $\alpha_{s,\tau,i}$ and the $\delta_{s,\tau,i}$. This is because the $h_{\tau,i}^e$ cannot be chosen in a way which eliminates the unknown Lagrange multipliers present in (4.2). Hence, estimating the parameters of the link between consumption and expenditures on the basis of (4.2), requires additional assumptions regarding these Lagrange multipliers.

Because of the difficulties with which one is confronted if one tries to estimate the model following the procedure desribed above, it could be worthwhile to consider an alternative approach for estimating model (3.1) on the basis of expenditure data only. This alternative consists of solving the model in two steps. In the first step, the model is maximized with respect to the consumption variables. Next, the optimal consumption bundle is written as a (vector-)function of the expenditure variables. After replacing the consumption variables by this function, a model which only depends on the expenditure variables results. In the second step this model is solved with respect to the expenditure variables. This second step model can then be used in estimation.

In order to study the working of this two step approach in greater detail, consider the following life cycle model which includes model (3.1) as a special case:[17]

$$\max_{c,e} \quad U(c)$$

$$\text{s.t. } \Phi(e,c) \in Z \qquad (4.3)$$

$$(c,e) \in C \times E$$

where

$U(\cdot)$ = expected lifetime utility function,

$C,\ E \subset L$,

L $\quad$ = set of all functions with domain V and range $\mathbb{R}^p$,

V $\quad$ = finite set of possible values of the vector of uncertainty inducing variables v (the so-called input variables),

c $\quad$ = p-dimensional vector containing consumption functions for each good in each period,

e $\quad$ = p-dimensional vector containing the expenditure functions for each good in each period,

$\Phi(\cdot)$ = q-dimensional vector of constraints on c and e,

Z $\quad$ = $\{z(\cdot) \in L;\ z(\cdot) \geq 0\} \subset L$.

After replacing the consumption variables in (4.3) by functions of expenditure variables the following second step model results:

$$\begin{aligned} &\max_{e} \quad U(F(e)) \\ &\text{s.t. } \Phi(e,F(e)) \in Z \\ &\qquad (F(e),e) \in C \times E \end{aligned} \tag{4.4}$$

where F: E → C, the (vector-)function relating the consumption variables to the expenditure variables; this function results from solving the first step (see the appendix for conditions under which the existence of this function is guaranteed).

An issue of particular interest is in what way this second step model is related to the models which are usually estimated, i.e., life cycle models of the type considered in chapter 2 which are (explicitly or implicitly) formulated in expenditure terms. Starting-point for the answer to this question is the model as given in (4.3), and in particular its objective function. Since this is an expected utility function, it can be written as follows:[18]

$$U(c) = \int_V u(c(v))\ dP(v) \tag{4.5}$$

where $u(\cdot)$: $\mathbb{R}^p \to \mathbb{R}$,

P = the probability distribution over V.

The objective function of the second step model results after replacing c by F(e)

$$U(F(e)) = \int_V u(F(e)(v))\ dP(v) \tag{4.6}$$

This function can be considered as an expected utility function if it can be written as follows:

$$U(F(e)) = \int_V \tilde{u}((e(v))\ dP(v) \tag{4.7}$$

for some $\tilde{u}$: $\mathbb{R}^p \to \mathbb{R}$

However, the function $u(F(e)(\cdot))$ whose expectation is determined in (4.6) will, in general, not be equal to the function $\tilde{u}(e(\cdot))$ in (4.7), as one must in (4.6) first apply the transformation $F(\cdot)$ to $e(\cdot)$. Loosely speaking, this implies that one needs the complete function $e(\cdot)$, and not just one possible value of this function, say $e(v)$, in order to be able to evaluate $u(F(e)(\cdot))$ in a particular point v. Notive that, in contrast, in order to evaluate $\tilde{u}(e(\cdot))$ in the same point v, one only needs to know $e(v)$, and not the complete $e(\cdot)$. Were this also the case with respect to $u(F(e)(\cdot))$, one could rewrite (4.6), using:

$$F(e)(v) = \tilde{F}(e(v))$$

for some $\tilde{F} \neq F$

which after substituting would result in

$$u(F(e)(v) = u(F(e(v))) = \tilde{u}(e(v))$$

where $\tilde{u} = u \circ F$

To illustrate that this procedure will, in general, not be applicable, consider the example depicted in Figures 1 and 2 where it is assumed that there is just one uncertainty inducing variable (so v is a scalar):[19]

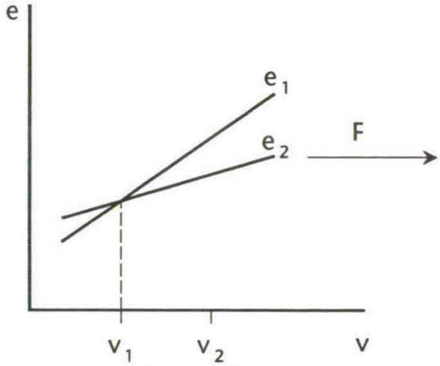

Figure 1

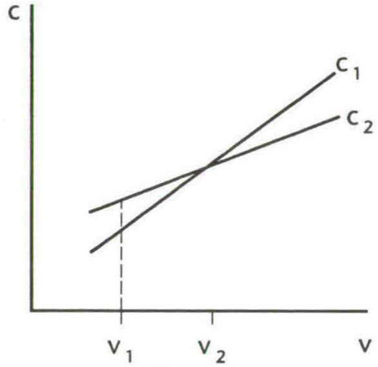

Figure 2

The functions $e_1(\cdot)$ and $e_2(\cdot)$, depicted in Figure 1, are elements of E, whereas $c_1(\cdot)$ and $c_2(\cdot)$, drawn in Figure 2, belong to the set C. The function which results from solving the first step, relating consumption to expenditures, establishes the link between both figures . It is assumed this function $F(\cdot)$ links $e_1(\cdot)$ to $c_1(\cdot)$, and $e_2(\cdot)$ to $c_2(\cdot)$. As can be seen from these figures, it is not sufficient to know the value of the expenditure functions at v_1 in order to be able to determine whether $c_1(v_1)$ $(=F(e_1(v_1)))$ or $c_2(v_1)$ $(=F(e_2(v_1)))$ corresponds with this value. For this, at least one additional value of each of the functions $e_1(\cdot)$ and $e_2(\cdot)$ is required, for example, those corresponding with v_2.

From the above it can be inferred that if one starts out from a life cycle model as given in (4.3), the model one ends up with in the second step will, in general, not be the familiar life cycle model which has an expected utility function as its objective function. So, if one believes that a model like the one given in (4.3) is an adequate description of the consumer's optimization problem, one cannot draw any conclusions regarding this model on the basis of results obtained from estimating a life cycle model in expenditure terms, in which an expected utility function is used as objective function. Instead one should use a model like the one given in (4.4). However, since solving this model requires knowledge of the shape of all the $e_j(\cdot) \in E$, this does not seem to be a straightforward task.

In summary, it can be concluded that both ways of estimating model (3.1) considered in this subsection, i.e., either directly from the first order conditions, or by using a two step approach, are applicable with great difficulty only, in the absence of consumption data.

4.2 The life cycle model under perfect foresight.

The problem with the two step approach discussed in the previous subsection was the fact that one needed to know all complete expenditure functions in order to be able to estimate the second step model. In case of the the life cycle model under perfect foresight, however, it is assumed that the consumer knows exactly which of the possible values of the vector of input variables v is realized. Hence, one only needs information on the value of the expenditure functions for this particular value of the input variables.

Given this feature, and the discussion in subsection 4.1, it is easily established that the objective function of the second stage model which results under the assumption of perfect foresight, is an (expected) utility function, implying that the second step model can be solved in the usual way. To demonstrate this, suppose that v_i is the actual realization of the input variables. The consumption and expenditure levels which are possible given this value v_i are $c_j(v_i)\ \forall\ c_j(\cdot) \in C$, and $e_j(v_i)\ \forall\ e_j(\cdot) \in E$, respectively. Since all other possible values of the consumption and expenditure functions are irrelevant for the consumer's optimization problem, as they will not occur, it is no longer necessary to know the complete consumption and expenditure functions. So, substituting F(e) for c in the (expected) utility function U(c) as defined in (4.5) now results in:

$$U(c) = u(c(v_i)) = u(F(e(v_i))(v_i)) = u(\tilde{F}(e(v_i))) = \tilde{u}(e(v_i)) \quad (4.8)$$

The function $\tilde{u}(\cdot)$ can be taken as an expected utility function, with a degenerated probability distibution of v, which concentrates all probability mass in the point v_i. Assuming v consists of just one variable, the consequences of the perfect foresight assumption can be illustrated by Figures 1 and 2. Suppose that v_1 is the value of the input variable which is realized. As Figure 1 shows, the corresponding values of the two expenditure functions are just a single point, and the two consumption functions in Figure 2 collapse to two points. Hence, the expenditure level is fully determined, and of the two possible consumption levels, the one resulting in the highest utility level is chosen. In order to make this choice, one needs, in contrast with the model under uncertainty, no information on the values of $c_1(\cdot)$ and $c_2(\cdot)$ for other possible realizations of v.

So, if one is willing to assume perfect foresight on behalf of the consumers, estimating a life cycle model in expenditure terms with an (expected) utility function as its objective function, can give some insight in the original model as given in (4.3). However, because of the following two reasons it is doubtful whether one can learn very much from these estimation results.

Firstly, it will in general be difficult to identify the parameters of the original model from the (reduced form) parameters of the second step model. Secondly, any model resulting in the second step cannot be distinguished from a properly chosen life cycle model in which the distinction between consumption and expenditures is ignored, and which is from the outset formulated in expenditure terms only.

4.3 The life cycle model under uncertainty in the presence of consumption data.

Given information on consumption, estimation of model (3.1) does not seem too difficult a task. In order to get some insight in how to derive a system of restrictions from condition (4.1) which can be used for estimation, three special cases of model (3.1) will be considered in this subsection. The first two cases are not particularly interesting in their own right, but they establish a link with the models considered in the previous chapters.

i) Only the lifetime budget constraint is binding:[20]

Under this assumption, condition (4.1) collapses to the following restriction:

$$E_t[\ \Sigma_{\tau=t}^{L}(\frac{1}{1+\rho})^{\tau-t}D_c u(c_\tau)'h_\tau^c - \lambda_t\cdot\Sigma_{\tau=t}^{L}(\frac{1}{1+r})^{\tau-t}p_\tau'h_\tau^e] = 0 \qquad (4.9)$$

On the basis of (4.9), one can derive many different sets of moment restrictions which can be used in estimation. For example, by choosing $h_{t,1}^c = 1/p_{t,1}$, $h_{t+1,1}^c = -(1+r)/p_{t+1,1}$, and all other $h_{\tau,i}^*$ (where * equals c or e) equal to zero, the Euler equation for the first good given in equation (4.2.5') of appendix B of chapter 2 results. Alternatively, if one has no information on the prices faced by consumers[21], condition (4.9) still allows one, in contrast with the first order conditions of the model used in chapter 2, to estimate the model, by setting, for instance, $h_{t,1}^c = 1$, and $h_{t+1,1}^c = -1$.

ii) Both the lifetime budget constraint and the nonnegativity constraints are binding:

In this case, condition (4.1) is reduced to the following restriction:

$$E_t[\ \Sigma_{\tau=t}^{L}(\frac{1}{1+\rho})^{\tau-t}D_c u(c_\tau)'h_\tau^c - \lambda_t\cdot\Sigma_{\tau=t}^{L}(\frac{1}{1+r})^{\tau-t}p_\tau'h_\tau^e + \Sigma_{\tau=1}^{L}\mu_\tau'h_\tau^c] = 0 \qquad (4.10)$$

Under the assumption that the nonnegativity constraints for the first good are not binding, and by a proper choice of the $h_{\tau,i}^*$, one can obtain the moment restrictions for the model with binding nonnegativity constraints considered in chapter 2. Choose $h_{t,1}^c = 1$, $h_{t,2}^c = -I_{(0,\infty)}(c_{t,2})$, and $h_{t+1,1}^c = -(1+r)\cdot(p_{t,1} - p_{t,2}\cdot I_{(0,\infty)}(c_{t,2}))/p_{t+1,1}$, set all other $h_{\tau,i}^*$ equal to zero, and restiction (4.2.8') in appendix B of chapter 2 results.

iii) Both the lifetime budget constraint and (some of) the upper bounds are binding:

For this case, the relevant part of condition (4.1) is:

$$E_t[\ \sum_{\tau=t}^{L}(\frac{1}{1+\rho})^{\tau-t}D_c u(c_\tau)'h_\tau^c - \lambda_t\cdot\sum_{\tau=t}^{L}(\frac{1}{1+r})^{\tau-t}p_\tau' h_\tau^e +$$

$$\sum_{\tau=1}^{L}\sum_{i=1}^{M}\nu_{\tau,i}\cdot[D_e R_{\tau,i}' h_i^e - D_c R_{\tau,i}' h_i^c]\ ] = 0 \tag{4.11}$$

To illustrate this case, set all $h_{\tau,i}^*$ equal to zero, except $h_{t,1}^c$ and $h_{t+1,1}^c$. After substituting these zeroes and the values of $D_e R_{\tau,1}$ and $D_c R_{\tau,1}$, the following restriction results:

$$E_t[\frac{\partial u(c_t)}{\partial c_{t,1}}\cdot h_{t,1}^c + (\frac{1}{1+\rho})\cdot\frac{\partial u(c_{t+1})}{\partial c_{t+1,1}}\cdot h_{t+1,1}^c + \nu_{t,1}h_{t,1}^c +$$

$$\nu_{t+1,1}(\delta_{t,t+1,1}h_{t,1}^c + h_{t+1,1}^c) + \ldots\ldots +$$

$$\nu_{L,1}(\delta_{t,L,1}h_{t,1}^c + \delta_{t+1,L,1}h_{t+1,1}^c] = 0 \tag{4.12}$$

By making some additional assumptions, this condition can be greatly simplified. For instance, under the assumption that the upper bound for period t is not binding, and that $\delta_{\tau,s,1} = \delta_1^{s-\tau}$ (i.e., a geometric decay structure), and by setting $h_{t+1,1}^c = -\delta_1\cdot h_{t,1}^c$, all unknown Lagrange multipliers drop out and the following condition results:

$$E_t\{[\frac{\partial u(c_t)}{\partial c_{t,1}} - (\frac{\delta_1}{1+\rho})\cdot\frac{\partial u(c_{t+1})}{\partial c_{t+1,1}}]\cdot h_{t,1}^c\} = 0 \tag{4.13}$$

which again shows some resemblance with the restrictions used in the previous chapters. But in contrast with those restrictions, condition (4.13) does not depend on prices or the interest rate. Instead, the parameter representing the durability aspect, i.e., δ_1, appears.

The special cases considered above indicate that, given information on consumption, a model like the one formulated in (3.1) can be estimated.[22] However, notice that the systems of restrictions derived as examples of these special cases, are by no means exhaustive. That is, conditions (4.9), (4.10), and (4.11) allow for many more (linearly independent) restrictions than those used in the examples (for instance, those in which not all $h^{e}_{\tau,i}$ are set equal to zero). This is of importance, since estimating such a subsystem is inefficient and might make it impossible to estimate all parameters of interest. Furthermore, the power of tests based on a subsystem of restrictions could be rather low.[23] This should be taken into account if one is in a situation allowing for the estimation of models of the type discussed here.

5. Concluding remarks.

In this chapter, the consequences for the life cycle model of taking into account the difference between expenditures and consumption were considered. Existing ways of dealing with this difference, i.e., the approaches used in infrequency of purchase models and in lag models, were discussed. As both types of models have their disadvantages, an alternative way of incorporating this difference in life cycle models was proposed.

Although this generalization seems attractive, the data requirements associated with it make it less suited for empirical applications. Put more precisely, the estimation of the model is straightforward only if one has information on the expenditures of households as well as on their consumption. Since information on consumption is very rarely available, one is forced to express the model in expenditure terms only to enable estimation.

Two approaches for achieving this aim were considered in this chapter. The first one starts from the first order conditions of the model in which the link between consumption and expenditures is incorporated, and by properly combining of these conditions, it removes all parts depending om consumption variables. However, this procedure makes it impossible to estimate the parameters characterizing the utility function.

Therefore, a second approach was considered in which the model was solved in two steps. In the first step one solves the model with respect to the consumption variables. After expressing the solution of this first step model in expenditure terms, a model depending on expenditure variables only results. This can then be solved and estimated in a second step. It was demonstrated that only if one assumes perfect foresight, is this second step related to the type of life cycle model which is usually estimated, i.e., a model formulated in expenditure terms only which has an (expected) utility function as its objective function.

But even if one restricts one's attention to the life cycle model under perfect foresight, one is still confronted with an important problem. That is, the second step model which results under this assumption cannot be distinguished from a properly chosen life cycle model in which the difference between consumption and expenditures is ignored, and which is from the outset formulated in expenditure terms only.

The overall conclusion which can be drawn from the above, is that if the difference between consumption and expenditures is considered to be important, and if one regards the already existing ways of dealing with this difference inadequately, not much insight can be gained from estimating a life cycle model in expenditure terms.[24] In order to be able to assess the importance of the difference between consumption and expenditures, it is necessary to collect consumption data next to expenditure data. Although it seems a difficult and expensive task to measure the consumption of households, the potential consequences of ignoring the difference between consumption and expenditures make research in this area more than necessary.

Appendix.

In order to formulate the second step model (4.4), it is necessary that c can be written as a function of e: $c = F(e)$. In this appendix, conditions guaranteeing the existence of such a function are given for the following two situations:

i) Conditions ensuring the uniqueness of the solution of the first step model hold.

Strict concavity of $U(\cdot)$, and convexity of the choice set with respect to c are sufficient to guarantee the uniqueness of the solution, and hence the existence of the function $F(\cdot)$.[25] To demonstrate this, assume that $\bar{c}$ and $\hat{c}$ both are solutions of the first step model:

$$U(\bar{c}) = U(\hat{c}) = U^{o} = \max_{c} \{U(c);\ \Phi(e,c) \in Z,\ (c,e) \in C{\times}E\} \qquad (A.1)$$

Because of the convexity of the choice set with respect to c, the linear combination $\alpha\bar{c} + (1-\alpha)\hat{c}$ with $\alpha \in (0,1)$, is also a possible solution of model (A.1). The strict concavity of $U(\cdot)$ implies that $U(\alpha\bar{c} + (1-\alpha)\hat{c}) > \alpha U(\bar{c}) + (1-\alpha)U(\hat{c}) = U^{o}$. Since this implies that U^{o} is not optimal, the first step model must have a unique solution, say c^{o}. So, model (A.1) has a unique solution for each value of e, which is just another way of saying that c^{o} is a function of e: $c^{o} = F(e)$.

ii) Conditions ensuring the uniqueness of the solution to the first step model do not hold.

In this situation, there is at each expenditure level more than one consumption level resulting in the optimum of the first step model. Hence, the optimal consumption is no longer a function of expenditure variables, but a correspondence. In Hildenbrand (1974), for example, sufficient conditions are given under which one can still express c as a function of e. Because these conditions are rather technical, the reader is referred to Hildenbrand (1974), page 54 for further details.

Notes to chapter 4.

1 The fact that the infrequency of purchase model is a static approximation of this underlying dynamic process was already pointed out by Blundell and Meghir (1987).

2 This is, for instance, likely to be the case for the vacation good, as one often has to pay (a part of) the vacation expenses well in advance. Ignoring this, can result in a consumption pattern which is to smooth.

3 To take account of this possible misspecification, some of the studies in this field (for example, Blundell and Meghir (1987), and Deaton and Irish (1984)) test the validity of the normality assumption. Alternatively, one could try to employ some semiparametric estimation procedure, thus making the normality assumption superfluous.

4 This assumption linking consumption to the available stock of a commodity is not always written down explicitly (see for example Pashardes (1986)). However, since consumers are usually assumed to derive utility (mainly) from consuming a good, and not from the fact that they possess a certain amount of it, such an assumption is necessary.

5 The aforementioned studies applying this 'lag approach', all use macro data. The only exception is Hayashi's study, in which quarterly household data are used. Given this lack of disaggregation, the subsequent discussion does have no bearing on the macro studies, and the relevance for Hayashi's work is limited.

6 Notice that the postponing of payments is not accounted for at all in the 'lag' model, since it links consumption only to past expenditures, not to future purchases.

7 Because habits refer to consumption, the lag polynomial linking consumption to expenditures does not represent habit formation, as claimed by, for example, Neusser (1988) and Muellbauer (1988).

8 Notice that there is another situation in which this will be the case, namely if the payments themselves are not delayed, but the reporting of them is. In most data sets, these two different mechanisms cannot be distinguished from each other.

9 An example of this situation could be the buying of clothes when the sales are on.

10 The upper bounds on consumption are constructed in such a way that total consumption does not exceeds total expenditures. For the special case in which all $\delta_{s,\tau}$ are set equal to one, this can be checked easily (simply consider last period's restriction).

11 Notice that these links only deal with the physical quantities which are bought or consumed, not with the costs associated with these activities.

12 The arguments of these functions, i.e., the consumption and expenditure variables, are suppressed for notational convenience.

13 This is possible if there exists a (second-hand) market for each good in which any quantity can be sold at the same price per unit which holds if one buys the particular good for new. It will be assumed that such markets exist.

14 This assumption results in a link between the two sets of parameters which is easy to represent. This in contrast with the original parameterization.

15 In the peak load pricing literature one sometimes comes across data sets containing information on the electricity use of households

(see, for example, Bartels and Fiebig (1990)). This can be seen as an example in which the consumption of a good, electricity, is observed.

16 The upper bounds all are written as nonnegativity constraints to facilitate the application of the Lagrange multiplier rule.

17 The representation of the life cycle model as given in (4.3), is based upon the formulation used by Melenberg and Alessie (1989).

18 Remember that c is a vector of (consumption) functions which depend on the uncertainty inducing variables v. Hence, in order to determine the expected utility one needs the probability distribution of v. For an extensive discussion on the technical aspects of the model as given in (4.1), the reader is referred to Melenberg and Alessie (1989).

19 The example is constructed for expository reasons only, and therefore kept as simple as possible.

20 The fact that none of the upper bounds is binding implies that one does not consume everything one bought during the lifetime. By assuming, for instance, that $\sum_{\tau=1}^{L} c_{\tau,i} \leq \sum_{\tau=1}^{L} e_{\tau,i}$ for i = 1,.., M are binding, this can be changed without greatly complicating the derivation of moment restrictions which can be used in estimation.

21 This is often the case in empirical work. In many studies information on national price indices must be used, since information on the prices consumers actually pay is lacking.

22 A much simpler version of model (3.1), which might be an interesting first step, results if one replaces the upper bounds on consumption by the following restriction: $\sum_{\tau=1}^{L} c_{\tau,i} \leq \sum_{\tau=1}^{L} e_{\tau,i}$, i = 1,..., M. This restriction simply states that one's lifetime consumption of each good cannot exceed the corresponding purchases made during this period.

23 Therefore, it is possible, for instance, that testing on the basis of such a subsystem leads to acception, whereas testing on the basis of

the full system of restrictions would result in rejection of the model.

24 It is even possible that the outcomes of such a model which are considered by the researcher to be favourable, would not be obtained if the estimation was repeated using consumption data.

25 Notice that these two conditions often are imposed in studies of the life cycle model.

CHAPTER 5

CONCLUSIONS

In this thesis, several aspects of the multi-good life cycle consumption model under uncertainty are considered. Starting-point for the analysis performed in this study is the multi-good version of Hall's (1978) life cycle model under uncertainty. Many life cycle models to be found in the literature can be seen as special cases of this multi-good version of Hall's model.

In chapter 2, the stochastic structure of this model is studied. It turns out that the first order conditions which must hold at the optimum imply intratemporal relations which are deterministic. As this implies that these relations must hold exactly for each observation of the dataset used in an application, which is unlikely to be the case, the presence of these deterministic relations implies that the model is misspecified.

The stochastic framework of the multi-good version of Hall's (1978) model must, therefore, be modified. Well-known ways of achieving this can be found in the literature, for instance, including measurement errors or random preferences in the model. However, since they suffer from some drawbacks, especially limiting the empirical applicability of these approaches, an alternative is considered: adding intratemporal uncertainty to the already present intertemporal uncertainty.

In chapter 2, the presence of this intratemporal uncertainty is motivated, and it is shown that this way of making the intratemporal relations non-deterministic allows for the estimation of quite general specifications of the life cycle model. The empirical usefulness of this approach is subsequently assessed by estimating two versions of the life cycle model with intratemporal uncertainty: a straightforward two-good version of Hall's (1978) model with intratemporal uncertainty, and a similar model with nonnegativity constraints which are binding. The results obtained for these models are, by and large, in accordance with consumer theory, and do not seem unfavourable to the generalization put forward in this chapter.

In chapter 3, the same multi-good life cycle framework is used to explain a consumption pattern one can observe for some goods. This pattern is characterized by the fact that such a good is either not consumed at all, or it is consumed in relatively large quantities only. It is argued that such a pattern cannot be fully explained by a straightforward multi-good version of Hall's (1978) model.

Therefore, a modification of this model which is designed to produce such a consumption pattern, is proposed. This is done by making either the preference ordering or the budget constraint non-convex for small values of the particular good displaying such a pattern. As a result of this change, it is never optimal to choose to consume small positive quantities.

In the empirical part of chapter 3, a life cycle model including an example of this transformation is estimated for a simple two-good case: vacation and non-vacation. The estimate of the parameter characterizing the transformation is large and positive, as it should be, but also imprecise.

It is put forward in chapter 3 that a possible explanation for this imprecise estimate could be that the data refer to expenditures, not consumption, and that the difference between these two quantities may be considerable. Therefore, the model is re-estimated twice on the basis of two datasets which result from two, rather simple, ways of taking this difference into account. However, in both cases the estimate of the parameter of the transformation remains imprecise.

So, it must be concluded that on the basis of the three datasets used in estimation, the model without the proposed modification cannot be rejected. In order to determine whether this result is due to specific features of the application considered in chapter 3, further research is needed. An interesting topic to be considered in this future work is the application of the modified life cycle model in a labour supply context. Typically, most people either do not work at all, or work a considerable number of hours, making this a good example of the pattern discussed in chapter 3. Moreover, since hours worked by an individual are more clearly defined, and more easily observed than the consumption of many goods, the data problems are likely to be considerably less. Hence, on the basis of the results of such a labour supply study, it is likely that one can draw more definite conclusions.

Chapter 4 is concerned with the problem touched upon in chapter 3, namely the consequences of the difference between consumption and expenditures for the multi-good life cycle consumption model. The way in which this problem was dealt with in chapter 3 was rather ad hoc. Chapter 4 is, therefore, devoted to a more thorough analysis. It starts by discussing the solutions to the problem put forward in the literature: the infrequency of purchase approach and the lag approach. However, since both of these approaches assume a rather restrictive link between consumption and expenditures, a more general model is formulated, which nests the lag models as special cases.

The question which is subsequently addressed in this chapter, is the empirical applicability of the more general model. This question is answered first for the situation with which one is most likely to be confronted, i.e., the situation in which one has information on the expenditures only. It is shown that for this case, estimating the model on the basis of the first order conditions, which is the usual way to obtain estimates of the parameters of interest, generally does not enable one to obtain estimates of the preference parameters.

Furthermore, it is shown that following an alternative approach, solving the model in two steps in such a way that the second step model depends on expenditure variables only, does not offer much improvement. This is because estimating the second step model requires, in general, information with respect to the complete lifetime.

If one, on the other hand, does have information on consumption, the model formulated in chapter 4 can be estimated. For some special cases, the system of equations on which this estimation can be based is derived.

The outcomes of this chapter, combined with the difficulties encountered in chapter 3, demonstrate the need for information on consumption. So, although it is not an easy exercise, it would be worthwhile to investigate how one could measure the actual consumption of households. One way to go about this could be to measure the stocks people keep. This information, combined with data on the purchases made in a certain period, can be used to calculate consumption levels. For some goods this procedure is rather easy to implement, as one can determine the size of the stock without great difficulty. An example of such a good is food. For other goods it might be possible to let consumers quantify the amount they keep in stock. Although such an approach may not be easy to implement, or perhaps more important to some, may not be not very cheap, it could give an indication of the importance of the difference between consumption and expenditures. On the basis of this information one can then assess the appropriateness of estimating the model which is usually considered, i.e., the life cycle model formulated from the outset in expenditure terms only.

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SAMENVATTING

Uitgangspunt van dit proefschrift is één van de modellen die een centrale plaats in de micro-econometrie innemen, namelijk het zogenaamde levenscyclus model. Meer in het bijzonder kunnen de modellen die in deze studie worden bestudeerd worden beschouwd als generalisaties van de meer-goerderen versie van Hall's (1978) levenscyclus model onder onzekerheid.

In hoofdstuk 2 ligt de nadruk op de stochastische specificatie van dit model. Dit, daar de eerste orde voorwaarden welke het optimale consumptie pad karakteriseren, intratemporele relaties impliceren welke deterministisch zijn. Het deterministische karakter van deze relaties betekent dat ze in empirische toepassingen exact moeten gelden voor iedere waarneming in de dataset die gebruikt wordt. Daar het zeer onwaarschijnlijk is dat men een schatbaar levenscyclus model kan afleiden dat aan deze eis voldoet, geeft de aanwezigheid van deze deterministische relaties een vorm van misspecificatie aan.

De in de literatuur gesuggereerde manieren om deze misspecificatie te verhelpen hebben hun tekortkomingen. In het bijzonder de empirische toepasbaarheid van deze benaderingen laat te wensen over. Daarom wordt een alternatieve wijziging van het model voorgesteld: de toevoeging van intratemporele onzekerheid naast de al aanwezige intertemporele. De aanwezigheid van deze intratemporele onzekerheid wordt gemotiveerd en enkele versies van het meer-goederen levenscyclus model met intratemporele onzekerheid worden geschat. De empirische resultaten zijn in het algemeen niet ongunstig.

In hoofdstuk 3 wordt bekeken in hoeverre het model geïntroduceerd in hoofdstuk 2 geschikt is om een bepaald consumptiepatroon te verklaren. Dit patroon wordt gekenmerkt door grote sprongen: men consumeert of helemaal niets van het desbetreffende goed, of men consumeert het slechts in (relatief) grote hoeveelheden. Een voorbeeld van zo'n goed is de vakantie van huishoudens.

Daar, zoals wordt aangetoond in hoofdstuk 3, zo'n patroon niet volledig verklaard kan worden met behulp van het gebruikelijke levenscyclus model, wordt een wijziging van dit model voorgesteld. Deze bestaat uit het zodanig veranderen van de preferentie ordening of de budgetrestrictie dat het niet optimaal is om te kiezen voor de consumptie van kleine hoeveelheden. Een nader uitgewerkte versie van dit gewijzigde model wordt vervolgens toegepast op het vakantie voorbeeld. De schatting van de parameter welke de voorgestelde wijziging karakteriseert is echter onnauwkeurig. Dit verandert niet indien rekening wordt gehouden, zij het op eenvoudige wijze, met het feit dat de data geen betrekking hebben op consumptie, maar op bestedingen.

Daar dit mogelijk onderscheid tussen deze twee grootheden toch een belangrijk probleem kan zijn voor de empirische toepasbaarheid van het meer-goederen levenscyclus model, wordt dit grondiger bestudeerd in hoofdstuk 4. Wederom wordt begonnen met een discussie van bestaande manieren om hiermee rekening te houden. Daar deze een nogal strikt verband veronderstellen, wordt een meer flexibel model geïntroduceerd. Vervolgens wordt de empirische toepasbaarheid van dit model onderzocht. Indien men alleen de beschikking heeft over bestedingsdata, blijkt deze toepasbaarheid gering. Afhankelijk van de gekozen methode kan men ofwel veel parameters niet schatten, of is hiervoor een in het algemeen niet voorhanden hoeveelheid data vereist. Indien men consumptie data tot zijn beschikking heeft, doen deze problemen zich niet voor en is het schatten van het in dit hoofdstuk geïntroduceerde model in het algemeen mogelijk.

In hoofdstuk 5 worden, ten slotte, de belangrijkste bevindingen van deze studie nogmaals kort op een rijtje gezet.